6.3 Graphs

```
#!/usr/bin/env python

import numpy as np
import scipy.sparse as sp

def sparseact(x, norm):
    return sp.csr_matrix(x > 0).toarray()

graphs = ...

# Debugging policy
```
Graphs as Matrices

- How could this (directed) graph be written as a matrix?

Weighted graphs: no problem
Undirected graphs: symmetric
Graph w/probability edges: columns sum to 1
\( \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix} \)
Matrices for Graph Traversal: Technicalities

- What is the general rule for turning a graph into a matrix?

- What does the matrix for an undirected graph look like?

- How could we turn a weighted graph (i.e., one where the edges have weights—maybe ‘pipe widths’) into a matrix?
Graph Matrices and Matrix-Vector Multiplication

- If we multiply a graph matrix by the $i$th unit vector, what happens?
Demo: Matrices for Graph Traversal
6.4 Sparsity
Storing Sparse Matrices

- Some types of matrices (including graph matrices) contain many zeros. Storing all those zero entries is wasteful. How can we store them so that we avoid storing tons of zeros?

\[
A = \{4; \{1: 0.4, 3: 0.2\}\}
\]
Storing Sparse Matrices Using Arrays

- How can we store a sparse matrix using just arrays? For example:

\[
\begin{pmatrix}
0 & 2 & 0 & 3 \\
1 & 4 & & \\
& & & \\
6 & 7 & \\
\end{pmatrix}
\]
Demo: Sparse Matrices in CSR Format
7 Norms and Errors
Norms

- What's a norm?
- Define norm.

Examples of Norms

- What are some examples of norms?

\[ \| (a_1) \|_2 = \sqrt{a_1^2 + b_1^2} \quad \text{2-norm} \]

\[ \| (a_1) \|_p = \sqrt[|p|]{|a_1|^p + |b_1|^p} \quad \text{p-norm} \]

\[ \| (1, 2) \|_3 = \sqrt[3]{8 + (27)} - 1 \]

\[ \| (x, y) \|_2 = \sqrt{x^2 + y^2} \]

- An "absolute value" for vectors
- $\mathbb{R}^n \rightarrow \mathbb{R}^p$
- $\| x \|_1$ if and only if:
  - $\| x \|_1 \geq 0$
  - $\| x \|_1 > 0 \iff x \neq 0$
  - $\| a \|_1 = \| a \| \cdot \| x \|_1$ For all numbers scalars $a$
Demo: Vector norms

$\|x-y\| \\
\|x\|, \|y\| \Rightarrow B/A/D
Norms and Errors

- If we’re computing a vector result, the error is a vector. That’s not a very useful answer to ‘how big is the error’. What can we do?