Overview

- CN
- What it’s good for: linear algebra
  interpolation

\[
\begin{pmatrix}
1 & 1 \\
-a_{21} & a_{11} \\
\end{pmatrix}
\begin{pmatrix}
a_{21} \\
a_{11} \\
\end{pmatrix} = 0
\]
\[ A = \sum_{i} m_i \]

Phina col second col for final col
Summary on Elimination Matrices

- El.matrices with off-diagonal entries in a single column just “merge” when multiplied by one another.
- El.matrices with off-diagonal entries in different columns merge when we multiply (left-column) * (right-column) but not the other way around.
- Inverse: Flip sign below diagonal
**LU Factorization**

- Can build a *factorization* from elimination matrices. How?
- Does this help solve $Ax = b$?

\[ A = LU \]

\[ L U x = 5 \]

\[ L y - 5 \leq \text{forward substitution } (n^2) \]

\[ U x = y \leq \text{backward substitution } (n^2) \]

With $LU$ in hand, solving $Ax - b$ costs $O(n^3)$. 
Demo: LU factorization

\[
\begin{pmatrix}
X \\
0
\end{pmatrix}
\begin{pmatrix}
\cdot \\
\cdot
\end{pmatrix} =
\begin{pmatrix}
\cdot \\
\cdot
\end{pmatrix}
\]

\[10^{-6} \leq u \leq 10^{-16}\]

output \quad \frac{10^{10}}{\text{comp}} \quad 10^{-16}

input
In-class activity: LU Factorization
LU: Failure Cases?

○ Is LU/Gaussian Elimination bulletproof?

\[
LU = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}
\]

\[
\begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} \begin{pmatrix} \frac{0}{0} \\ \frac{1}{1} \end{pmatrix}
\]

\[u_{11} \cdot 1 = 0 \Rightarrow u_{11} = 0\]

\[l_{21} \cdot u_{11} + 1 \cdot 0 = 2\]

○ What can be done to get something *like* an LU factorization?

Swap some rows
Fixing nonexistence of LU

- How do we capture ‘row switches’ in a factorization?

- What does this process look like then?

By absolute value

"partial pivoting"

ideally take the biggest entry in the column and swap it onto the diagonal

this keeps the factor in the elimination matrix as small as possible

less roundoff error
Any permutation matrix (not just row swaps) $P$ can be inverted as $P^{-1}$.

$$M_3 \theta_3 M_2 \theta_2 M_1 P_2 M_1 P_1 A = U$$

$$A = P^T M_1^{-1} P_1^T M_2^{-1} P_2^T M_3^{-1} U$$

Down triangular? No!

For all $15$ permutations $P^T A = CH$
Computational Cost

- What is the computational cost of multiplying two $n \times n$ matrices?

- What is the computational cost of carrying out LU factorization on an $n \times n$ matrix?
Demo: Complexity of Mat-Mat multiplication and LU