Overview

- Eigenvalues

- What ev are good for

- SVD / lSqr
Power Iteration

- What are the eigenvalues of $A^{1000}$?

Assume $|\lambda_1| > |\lambda_2| > \cdots > |\lambda_n|$ with eigenvectors $x_1, \ldots, x_n$.
Further assume $\|x_i\| = 1$. 

\[ A^{1000} y = \alpha_1 x_1 + \cdots + \alpha_n x_n \]

\[ A \frac{\lambda}{1000} x_i = \lambda x_i \]
\[(\lambda - 1.75)^{-1} \quad \text{power method that}
\]

The power method applied to \((A - \sigma I)^{-1}\) converges to the eigenvector (of \(A\)) that's closest to \(\sigma\).

Inverse iteration

Can also choose the shift to be the Rayleigh quotient.
Power Iteration: Issues?

○ What could go wrong with Power Iteration?
What about Eigenvalues?

- Power Iteration generates eigenvectors. What if we would like to know eigenvalues?
Convergence of Power Iteration

- What can you say about the convergence of the power method?
  Say \( v_1^{(k)} \) is the \( k \)th estimate of the eigenvector \( x_1 \), and

\[
e_k = \| x_1 - v_1^{(k)} \|.
\]

Possible:

\(|\lambda_2| = |\lambda_1|\)

\(\Rightarrow\) power method won't converge

\(\lambda_1 = 3\)

\(\lambda_2 = -3\)

addressed by a shift
Inverse Iteration / Rayleigh Quotient Iteration

○ Describe inverse iteration.

○ Describe Rayleigh Quotient Iteration.
**Demo:** Power Iteration and its Variants

**In-class activity:** Eigenvalue Iterations
Computing Multiple Eigenvalues

- All Power Iteration Methods compute one eigenvalue at a time. What if I want all eigenvalues?

\[
\text{Deflation: } A \tilde{v} = \lambda \tilde{v} \\
V = \{ \alpha \tilde{v} : \alpha \in \mathbb{R} \}
\]

\[
A : V \rightarrow V \\
A : V^\perp \rightarrow V + V^\perp
\]

\[
A = \begin{pmatrix}
\lambda_1 & \cdots & 0 \\
0 & \ddots & \vdots \\
0 & \cdots & \lambda_n
\end{pmatrix}
\]

Eigenvalues of \(A\): \(\lambda_i\) \(U\) eigenvalues of \(B\)
Simultaneous Iteration

- What happens if we carry out power iteration on multiple vectors simultaneously?

\[ A^k x \rightarrow \text{all columns go toward the same vector} \]

Idea: keep them different (at every iteration) using Gram-Schmidt.
11 Eigenvalues: Applications
Markov chains

- Consider the following graph of states:

Suppose this is an accurate model of the behavior of the average student. :)
How likely are we to find the average student in each of these states?