Shifted Tayor expansion

$f(x) = 1 - 3x + x^2$

$$f(x) = \frac{f(a)}{0!}(x-a)^0 + \frac{f'(a)}{1!}(x-a)^1 + h.o.t.$$

$$\approx f(3)(-1)^0 + f'(3)(-1)$$

$$= f(3) - f'(3)$$

$$f(3) = 1 - 3(3) + 3^2 = 1$$

$$f'(3) = -3 + 2x\Big|_{x=3} = -3 + 6 = 3$$

$$\boxed{f(3) \approx 1 - 3 = -2}$$

Distributron function

$$\int_0^{10} \frac{1}{c} * (3)t^2 = 1 \Rightarrow c = \int_0^{10} 3t^2$$

$$= t^3\Big|_{10} - t^3\Big|_0 = 1000$$

Interpolating function

The accuracy of the interpolating function depends on

a) The function being interpolated

b) The interpolatoon points

Expectation value

d4: $\frac{1+2+3+4}{4} = \frac{10}{4} = \frac{5}{2} = 2.5$

d8: $\frac{1+2+3+4+5+6+7+8}{8} = \frac{36}{8}$

$= \frac{18}{4} = \frac{9}{2} = 4.5$

$(2.5)3 + 4.5(2) = 9 + 7.5 = 16.5$ damage

Vandermonde

$$\begin{bmatrix} 1 & -x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -27 \\ -1 \\ 0 \end{bmatrix}$$

You will not be asked to solve by hand on the examlet

$p(x) = -1 + 5(t) - 4t^2$

$\cancel{1 + 2 + 3 + 4 + 5 + 6}$

$\cancel{3 \quad 7 \quad 11}$

$\cancel{10 \quad \frac{21}{6}}$

Interpolation error

An $(n-1)$ degree polynomial results in $O(h^n)$ error.

Here $n = 4$. With a factor of 3 reduction in step size we expect a factor of $3^4 = 81$ reduction in error.

expected error $= \frac{0.1}{81} = 0.01234$

random variables and monte carlo