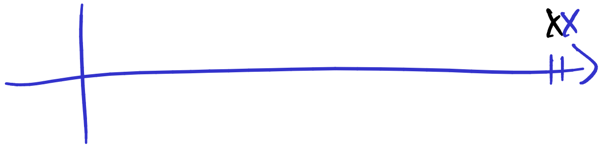
IEEE Overview

- Cancellation
- Vectors
- Matrices
- Norms

Problems with FP Addition

- What happens if you subtract two numbers of very similar magnitude? As an example, consider $a = (1.1011)_2 \cdot 2^0$ and $b = (1.1010)_2 \cdot 2^0$.



$$a = (1.011)_2 \cdot 2^0 \leftarrow \text{rel. error: mach. } \varepsilon \quad 0.001 = 2^{-3}$$

$$b = (1.010)_2 \cdot 2^0$$

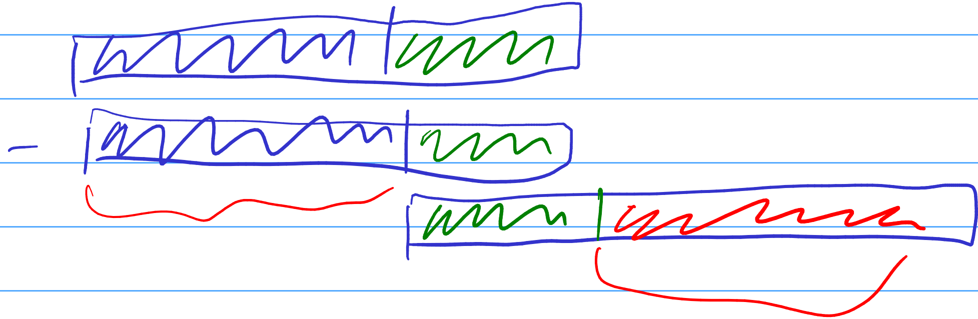
$$(a-b) = (0.001\,\_\,\_\,\_)\cdot 2^0$$

$$= (1.\underline{0}\,\underline{0}\,\underline{0}) \cdot 2^{-3} \quad ?$$

Demo

rel. error

$$\frac{(1.111)_2 \cdot \cancel{2^{-3}} - (1.000)_2 \cdot \cancel{2^{-3}}}{(1.000)_2 \cdot \cancel{2^{-3}}} = \frac{(1.111)_2 - (1.000)_2}{1} = \frac{(0.111)_2}{1} \approx 87\%$$



Part 2:
Arrays–Computing with Many Numbers

6 Modeling the World with Arrays

6.1 The World in a Vector

Some Perspective

- We have so far (mostly) looked at what we can do with single numbers (and functions that return single numbers).
- Things can get *much* more interesting once we allow not just one, but *many* numbers together.
- It is natural to view an *array of numbers* as one object with its own rules.
  The simplest such set of rules is that of a vector.
- A 2D array of numbers can also be looked at as a matrix.
- So it's natural to use the tools of computational linear algebra.
- 'Vector' and 'matrix' are just *abstract structures* that come to life in many (*many!*) applications. The purpose of this section is to explore some of those applications.

Vectors

- What's a vector?

x, y: vector in a vector space

- $x + y$
- αx
- $\alpha(x+y) = \alpha x + \alpha y$

... (some same rules)

Vectors from a CS Perspective

- What would the concept of a vector look like in a programming language (e.g. Java)?

```
interface Vector {
    Vector add( Vector x, Vector y);
    Vector mul ( Number alpha, Vector x));
}
```

Vectors in the 'Real World'

Demo: Images as Vectors
Demo: Sound as Vectors
Demo: Shapes as Vectors

6.2 What can Matrices Do?

Matrices

$f(x+y)=f(x)+f(y)$

$f(\alpha x)=\alpha f(x)$

$f : U \longrightarrow V$

$u_1 \ldots u_n \qquad v_1 \ldots v_m$

What does a matrix do?

It represents a *linear function* between two vector spaces $f \colon U \to V$ in terms of bases $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n$ of U and $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_m$ of V. Let

$$\boldsymbol{u} = \alpha_1 \boldsymbol{u}_1 + \cdots + \alpha_n \boldsymbol{u}_n$$

and

$$\boldsymbol{v} = \beta_1 \boldsymbol{v}_1 + \cdots + \beta_m \boldsymbol{v}_m .$$

Then f can *always* be represented as a matrix that obtains the βs from the αs:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix} .$$

Example: The 'Frequency Shift' Matrix

- Assume both $\boldsymbol{u}$ and $\mathbf{v}$ are linear combination of sounds of different frequencies:

$$\boldsymbol{u} = \alpha_1 \boldsymbol{u}_{110\text{ Hz}} + \alpha_2 \boldsymbol{u}_{220\text{ Hz}} + \cdots + \alpha_4 \boldsymbol{u}_{880\text{ Hz}}$$

(analogously for $\boldsymbol{v}$, but with βs). What matrix realizes a 'frequency doubling' of a signal represented this way?

$$\vec{\alpha} = (\alpha_1, \ldots, \alpha_4)$$

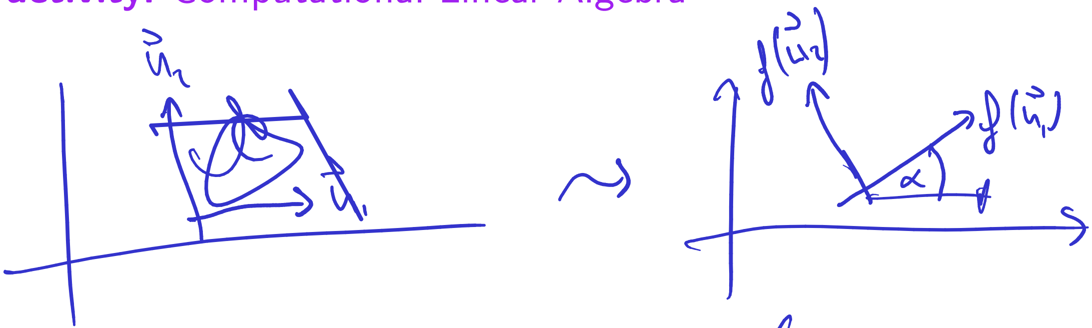
$$A\vec{\alpha} = (0, \alpha_1, \ldots, \alpha_3)$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

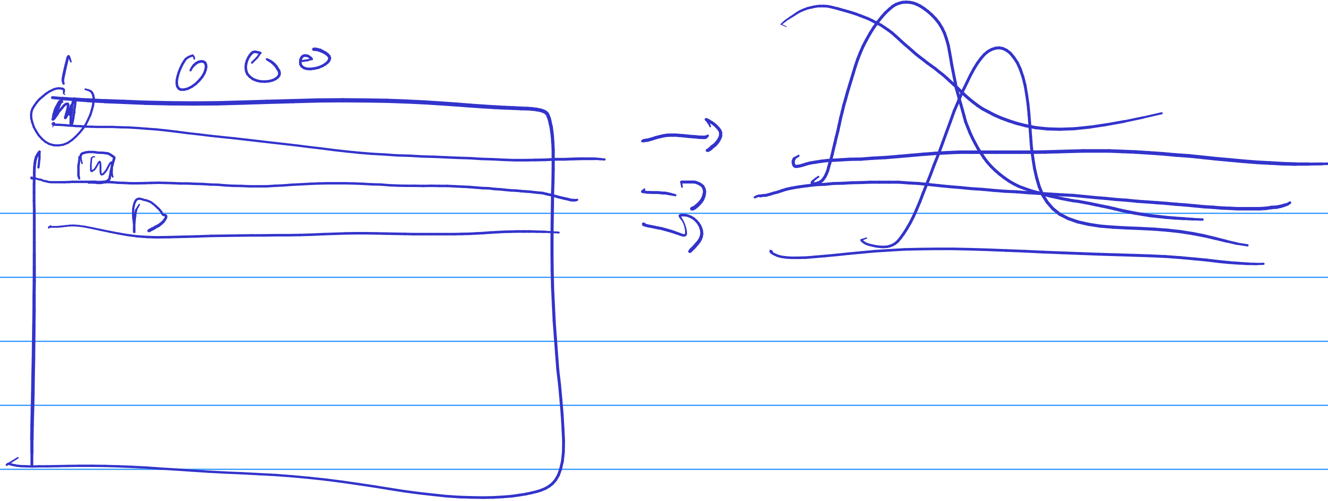
Matrices in the 'Real World'

What are some examples of matrices in applications?

Demo: Matrices for Geometry Transformation
Demo: Matrices for Image Blurring
In-class activity: Computational Linear Algebra



$$A \quad \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ y_1 & y_2 & \cdots & y_n \end{pmatrix}$$



6.3 Graphs