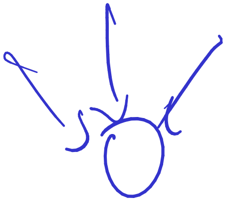
6.3 Graphs

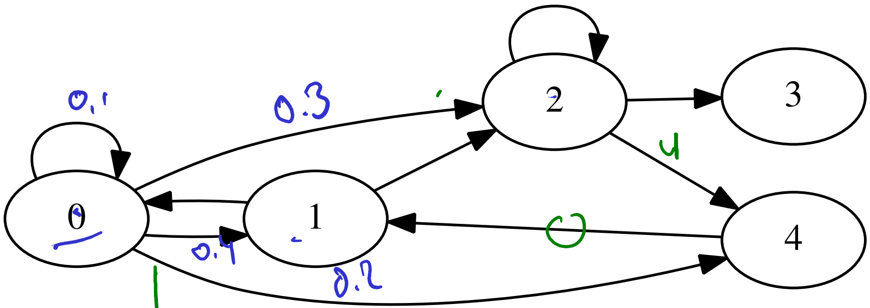
→ Debugging policy

- graphs
- sparse mat.
- norm

Graphs as Matrices

- How could this (directed) graph be written as a matrix?





$$\begin{array}{c|c}
 & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\
\hline
\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 1 & 1 & & & \\ 1 & 0 & & & \\ 4 & 1 & & & \\ 0 & 0 & & & \\ 1 & 0 & & & \end{pmatrix}
\end{array}$$

0

weighted graphs: no problem

undirected graphs: symmetric.

graph w/ probability edges: columns sum to 1

$\Delta x = \_\ x$

Matrices for Graph Traversal: Technicalities

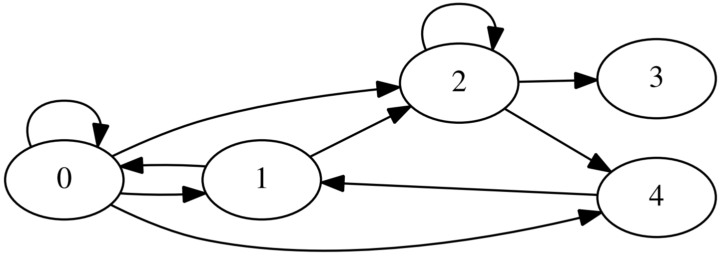
- What is the general rule for turning a graph into a matrix?

- What does the matrix for an *undirected* graph look like?

- How could we turn a *weighted graph* (i.e. one where the edges have weights–maybe ‘pipe widths’) into a matrix?

Graph Matrices and Matrix-Vector Multiplication

- If we multiply a graph matrix by the ith unit vector, what happens?



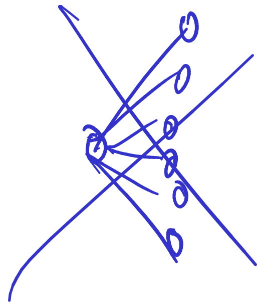
Demo: Matrices for Graph Traversal

6.4 Sparsity

Storing Sparse Matrices

- Some types of matrices (including graph matrices) contain many zeros.
  Storing all those zero entries is wasteful.
  How can we store them so that we avoid storing tons of zeros?

$A = \{4: \{1: 0.4,\ 3: 0.2\}\}$



Storing Sparse Matrices Using Arrays

- How can we store a sparse matrix using just arrays? For example:

$$\begin{pmatrix} 0 & 2 & 0 & 3 \\ 1 & 4 & & \\ & & \cancel{5} & \\ 6 & & & 7 \end{pmatrix}$$

Compressed
Sparse
Row

RowStarts = [0 2 4 5 7]

ColIndices = [1 3 0 1 2 0 3]

Values = (2 3 1 4 5 6 7)

RowStarts = (0 2 4 4 6)

ColIndices = [1 3 0 1 0 3]

Values = (2 3 1 4 6 7)

Demo: Sparse Matrices in CSR Format

7 Norms and Errors

Norms

- What's a norm?

- An "absolute value" for vectors
- $\mathbb{R}^n \to \mathbb{R}^+_0$
- $\|\vec{x}\|$

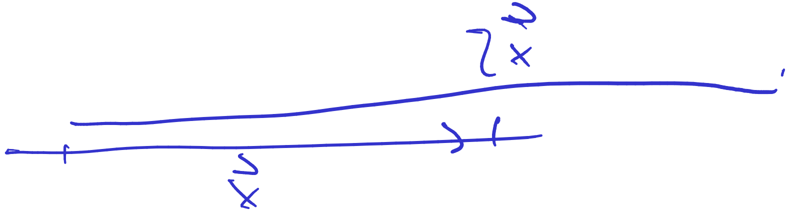
$$\begin{pmatrix}1\\2\\3\end{pmatrix} \qquad \|\vec{x}\|$$

- Define norm.

$\|\vec{x}\|: \mathbb{R}^n \to \mathbb{R}^+_0$

If and only if:

- $\|\vec{x}\| \geq 0$
- $\|\vec{x}\| > 0 \iff \vec{x} \neq 0$
- $\|\alpha\vec{x}\| = |\alpha|\,\|\vec{x}\|$ For all numbers scalars α
- $\|\vec{x}+\vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$



Examples of Norms

- What are some examples of norms?

$$\left\|\begin{pmatrix}a\\b\end{pmatrix}\right\|_2 = \sqrt{a^2+b^2} \qquad \text{2-norm}$$

$$\left\|\begin{pmatrix}a\\b\end{pmatrix}\right\|_p = \sqrt[p]{|a|^p+|b|^p} \qquad \text{p-norm}$$

$$\left\|\begin{pmatrix}2\\-3\end{pmatrix}\right\|_3 = \sqrt[3]{8+(-27)}$$

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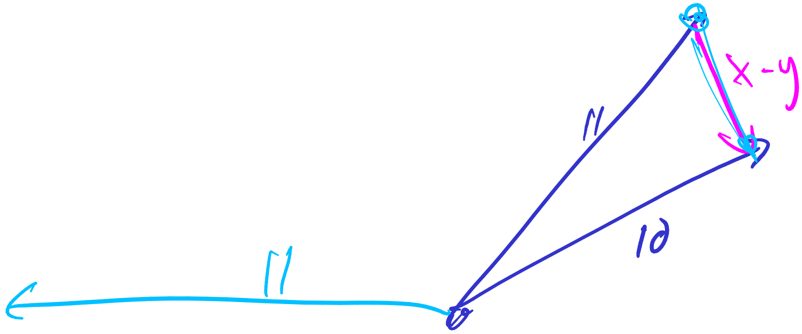
Demo: Vector norms



BAD: $\|x\| - \|y\|$

$\|\vec{x} - \vec{y}\|$

Norms and Errors

- If we're computing a vector result, the error is a vector.
  That's not a very useful answer to 'how big is the error'.
  What can we do?