~~Taylor of order 4 → Taylor of order 6~~

Taylor series of order n:

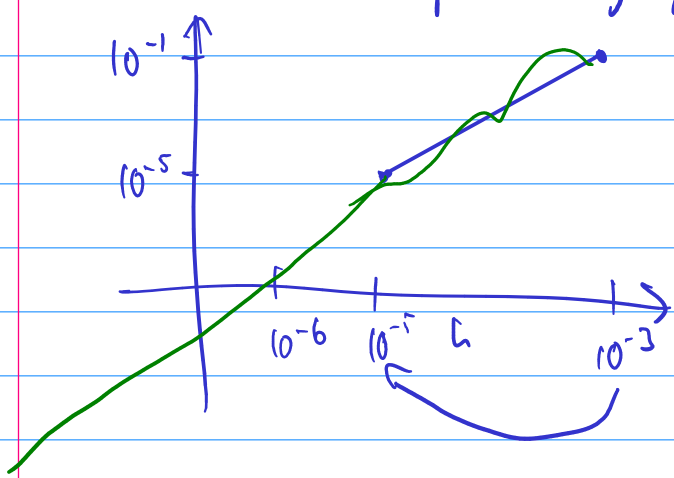
$$f(c) + f'(c)(x-c) + \frac{f''(c)}{2}(x-c)^2 + \dots \frac{f^{(n)}(c)}{n!}(x-c)^n$$

$$\underbrace{+ \frac{f^{(n+1)}(c)}{(n+1)!} \underbrace{(x-c)^{n+1}}_{|\cdot| \le h}}_{\sim \text{Error}}$$

$$\underbrace{\text{Error}(h) = O(h^{n+1})}$$

captures dependency on h

captures asymptotic behavior as $h \to 0$



$O(h^2)$ ↗ $\text{Error}(h) \le C \cdot h^2$

$\text{Error}(h \cdot 10^{-2}) \le C \cdot 10^{-4} \cdot h^2$

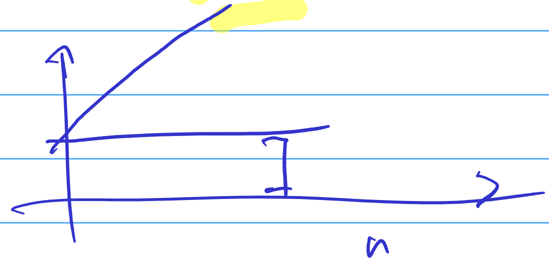
$\le 10^{-4} \cdot \text{Error}(h)$

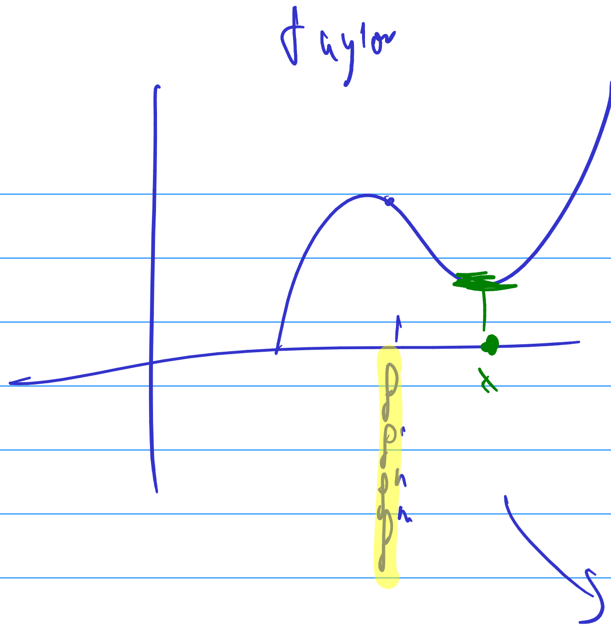
LU Factorization has a runtime of $O(n^3)$ for an $n \times n$ mat.

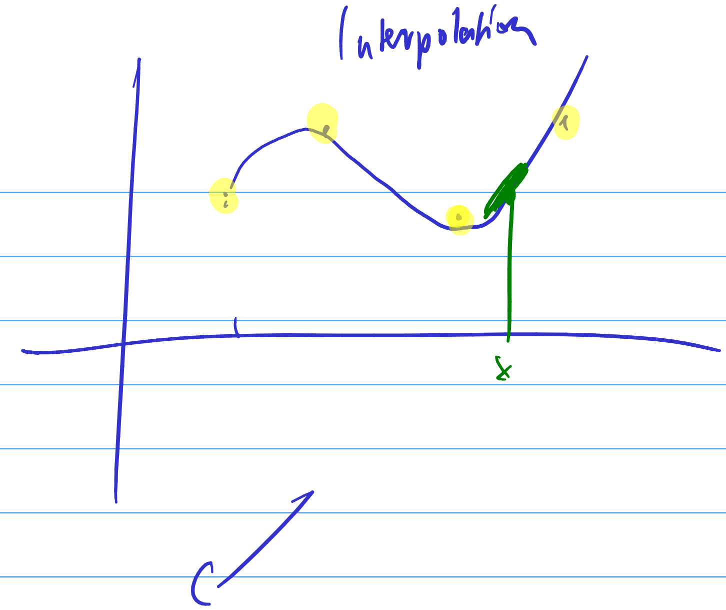
$\text{Time}(1000) = 2s$ $\qquad\qquad$ $\text{Time}(n) \le C \cdot n^3$

$\text{Time}(2000) = C \cdot 2^3 \cdot 1000^3 = 2^3 \cdot \text{Time}(1000)$

$2 \cdot 1000$

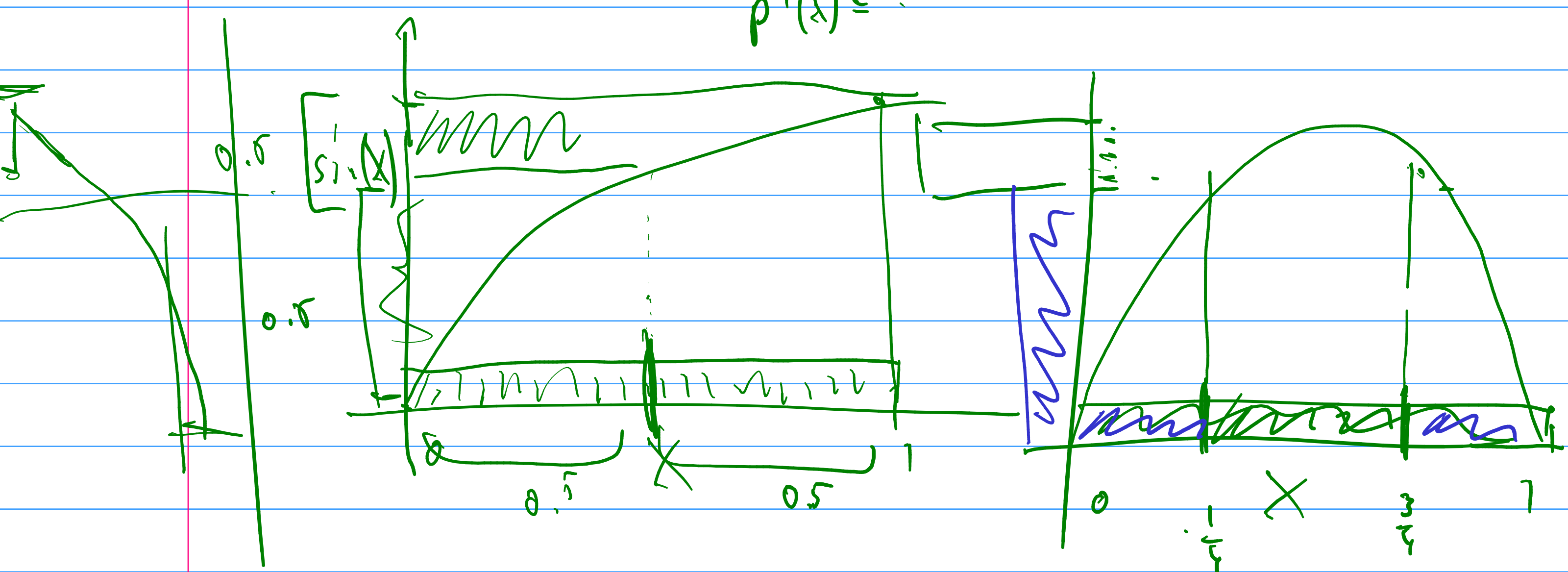


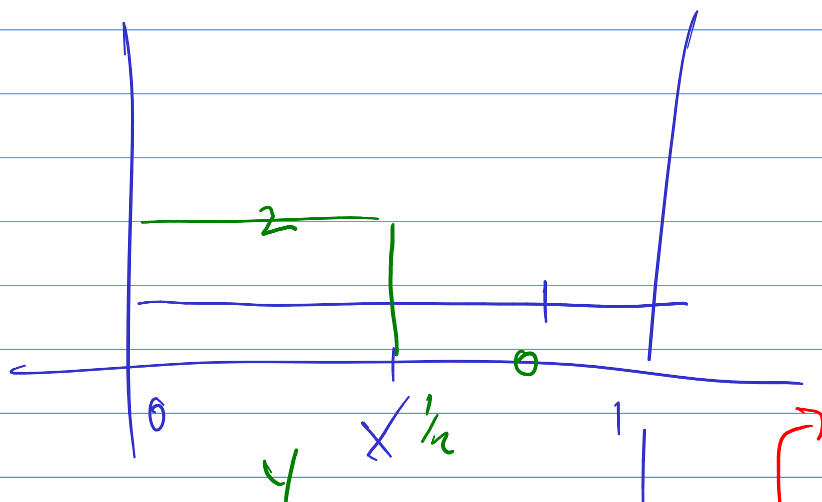




$\rightarrow$ $p(x) = x^3 + 2x^2 + 10x - 5$

$p'(x) =$





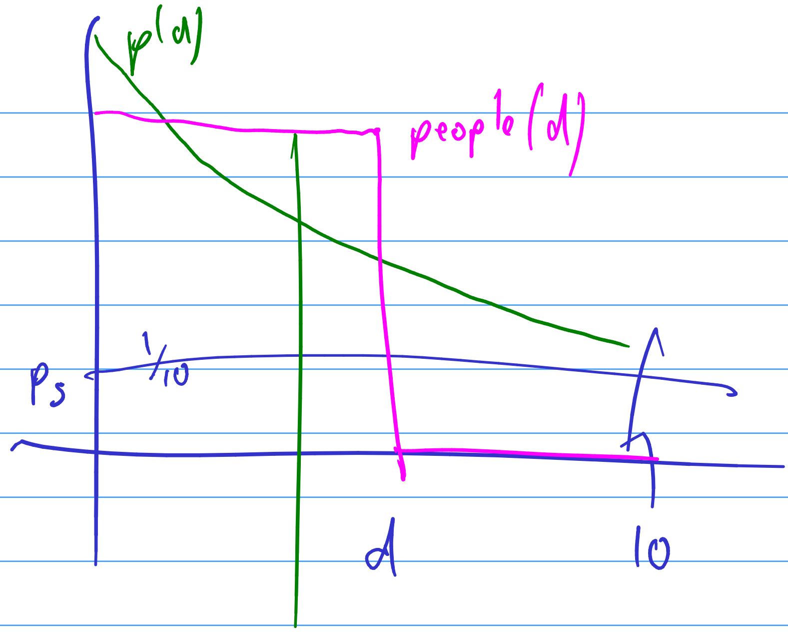
happy accident

$$E[\sqrt{Y}] = \frac{1}{N}\sum_{i=1}^{N} \sqrt{y_i}$$

where y_i have to be distributed like Y

$y_i = x_i/2$ ← one way to make samples that have the right dist.

$$\hookrightarrow \frac{1}{N}\sum_{i=1}^{N} \sqrt{x_i} \cdot \frac{p_Y(x_i)}{p_X(x_i)}$$



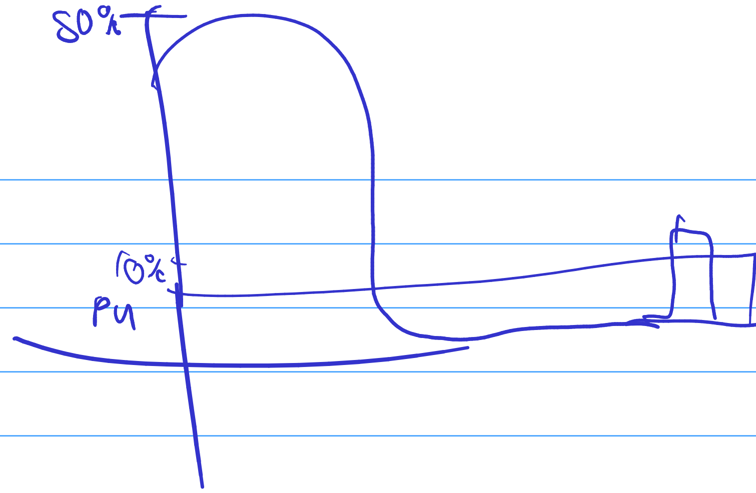
$s_i \sim p_s$

$E[people(d)]$

$= \frac{1}{N} \sum_{i=1}^{N} people(s_i) \cdot \frac{p(s_i)}{p_s}$

$= \frac{10}{N} \sum_{i=1}^{N} people(s_i) \cdot p(s)$

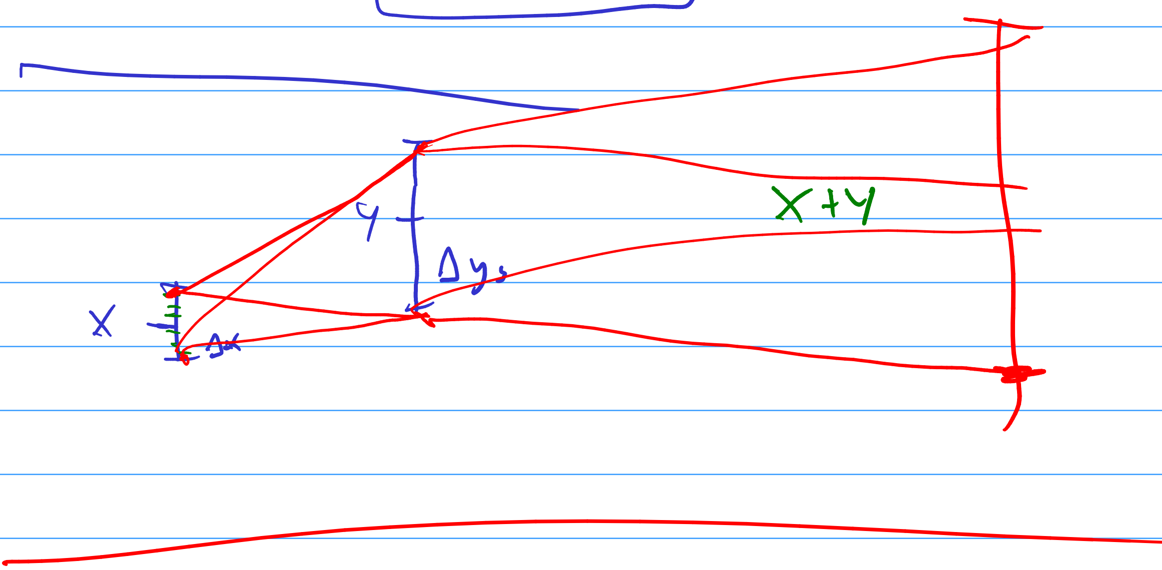
$$p(x) = C \cdot \hat{p}(x)$$



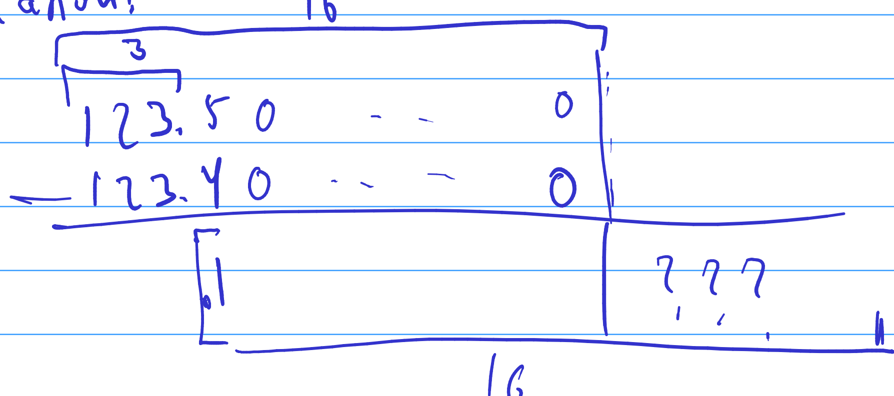
$$1 = E_p[1] = E_u\left[1 \cdot \frac{p(u)}{p_u(u)}\right]$$

$$= E_u\left[1 \, \frac{C \cdot \tilde{p}(u)}{p_u(u)}\right]$$

$$= \underbrace{C \cdot \ldots \text{ sample mean}}$$



Cancellation:



retained digits in cancellation = shared digits − identical at front.

Have samples x_i distributed like $p_X(\ldots)$.

Can: $E[f(X)] \approx \frac{1}{N} f(x_i)$

$\hookrightarrow \int_{\mathbb{R}} f(X) \cdot p_X(X)\, dX$

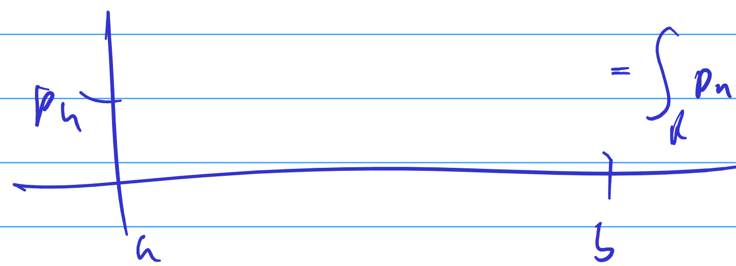
$E[\sqrt{Y}]$

$= E\left[\sqrt{X} \cdot \frac{p_Y(X)}{p_X(X)}\right]$

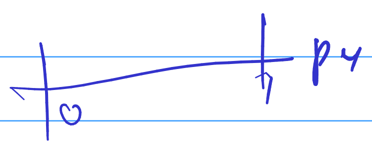
$= \int \sqrt{x}\, \frac{p_Y(x)}{p_X(x)} \cdot p_X(x)\, dx$

$= \frac{1}{N} \cdot \sum \sqrt{x_i} \cdot \frac{p_Y(x)}{p_X(x)}$

$\overbrace{\underbrace{1.1111}_{5}0 \quad 1.11111}^{83}$



$= \int_{\mathbb{R}} p_n = 1$



$\frac{p_Y}{p_X = \frac{1}{10}}$