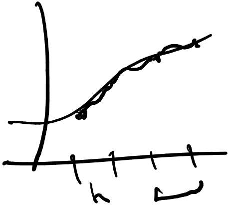
Why polynomials?

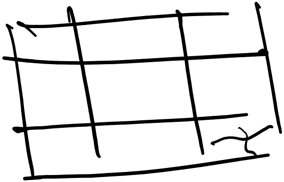
$$a_3x^3 + a_2x^2 + a_1x + a_0$$

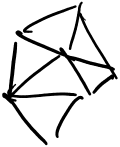
- How do we write a general degree n polynomial?

$$\sum_{i=0}^{n} a_i x^i = a_0 x^0 + a_1 x + a_2 x^2, \dots$$

- *Why* polynomials and not something else?







Reconstructing a Function From Derivatives

- If we know $f(x_0)$, $f'(x_0)$, $f''(x_0)$, can we approximately reconstruct the function as a polynomial p?

$$p(x) = ??? + ???x + ???x^2 + \cdots$$

$f(0), f'(0), f''(0)$

$p(x) = a + bx + cx^2 + dx^3 + \ldots$

$p(0) = a$

$p'(x) = b + 2cx + 3dx^2$

$p'(0) = b$

$p''(x) = 2c + 6dx + \ldots$

$c = p''(0)/2$

$$g(x) = \sum_{i=0}^{\infty} \frac{g^{(i)}(0)}{i!} x^i$$

$$g(x) = f(x + x_0)$$

Demo: Polynomial Approximation with Derivatives (Part I)

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(x_0)}{i!} (x - x_0)^i$$

Shifting the Expansion Center

- Can you do this at points other than the origin?

$$g(x) = f(x + x_0)$$