Overview

Announcements

HW 4

Examlet 2

Vectors

What's a vector?

Vectors in the 'Real World'

Demo: Images as Vectors
Demo: Sound as Vectors
Demo: Shapes as Vectors ✓

Outline

Matrices

What does a matrix do?

It represents a *linear function* between two vector spaces $f : U \to V$ in terms of bases $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n$ of U and $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_m$ of V. Let

$$\boldsymbol{u} = \alpha_1 \boldsymbol{u}_1 + \cdots + \alpha_n \boldsymbol{u}_n$$

and

$$\boldsymbol{v} = \beta_1 \boldsymbol{v}_1 + \cdots + \beta_m \boldsymbol{v}_m.$$

Then f can *always* be represented as a matrix that obtains the βs from the αs:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix}.$$

Example: The 'Frequency Shift' Matrix

Assume both $\boldsymbol{u}$ and $\mathbf{v}$ are linear combination of sounds of different frequencies:

$$\boldsymbol{u} = \alpha_1 \boldsymbol{u}_{110\text{ Hz}} + \alpha_2 \boldsymbol{u}_{220\text{ Hz}} + \cdots + \alpha_4 \boldsymbol{u}_{880\text{ Hz}}$$

(analogously for $\boldsymbol{v}$, but with βs). What matrix realizes a 'frequency doubling' of a signal represented this way?

Matrices in the 'Real World'

What are some examples of matrices in applications?

Demo: Matrices for Geometry Transformation
Demo: Matrices for Image Blurring
In-class activity: Computational Linear Algebra

Outline

Graphs as Matrices

How could this (directed) graph be written as a matrix?