

Overview

- FP
- Inclass
-

- IEF debrief (nextVue)
- Exanlet 2
(extended!)
- HW4 (out soon)

$$1.23 \cdot 10^2$$

1.2345

Unum



Universal number

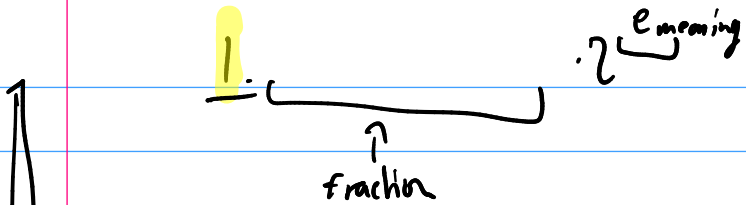
$$(1.1011)_2 \cdot 2^5$$

$$+ 1.0010 \cdot 2^3$$

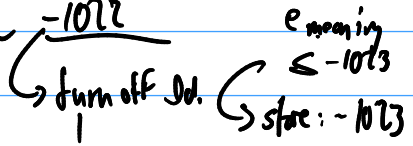
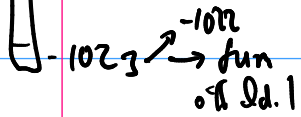
$$\rightarrow + 0.0100 | 10 \cdot 2^5$$

$$+ 1.1011 \cdot 2^5$$

$$10.0000$$



$$e_{\text{meaning}} = \begin{cases} -1073 + e_{\text{stored}} & e_{\text{meaning}} > -1073 \\ -1072 & e_{\text{meaning}} \leq -1073 \end{cases}$$



Demo: Density of Floating Point Numbers

Demo: Floating Point vs. Program Logic

Floating Point and Rounding Error

What is the relative error produced by working with floating point numbers?

What is smallest floating point number > 1 ? Assume ~~3~~ stored bits in the significand.

$$x = 1.\underbrace{001}_{\uparrow 2^{-3}} \cdot 2^0$$

$$\frac{x-1}{1} = 2^{-3} = 2^{-3}$$

What's the smallest FP number > 1024 in that same system?

$$y = 1.\underbrace{001}_{\uparrow 2^{-3}} \cdot 2^{10}$$

$$\frac{y-1024}{1024} = \frac{1024 - 2^{10}}{2^{10}} = \frac{2^{10} - 2^{10}}{2^{10}} = 2^{-3}$$

Can we give that number a name?

value of last digit of significand:
machine epsilon

What does this say about relative error?

Relative error in rounding to
FP: machine epsilon

Implementing Arithmetic

How is floating point addition implemented?

Consider adding $a = (1.101)_2 \cdot 2^1$ and $b = (1.001)_2 \cdot 2^{-1}$ in a system with three bits in the significand.



Demo: Floating point and the harmonic series

$$\sum_{i=1}^{\infty} \frac{1}{i}$$

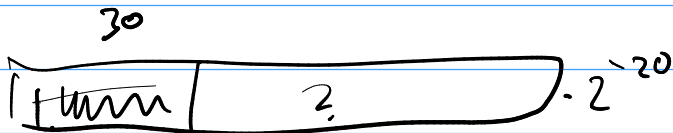
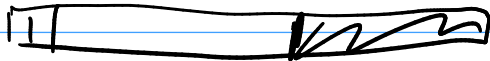
1. _____ 2^{exp}

Problems with FP Addition

What happens if you subtract two numbers of very similar magnitude?

As an example, consider $a = (1.1011)_2 \cdot 2^0$ and $b = (1.1010)_2 \cdot 2^0$.





$(1. \quad)_2$ wrong

Demo: Catastrophic Cancellation

In-class activity: Floating Point 2

Outline

Python, Numpy, and Matplotlib
Making Models with Polynomials
Making Models with Monte Carlo

Error, Accuracy and Convergence
Floating Point

Modeling the World with Arrays

The World in a Vector
What can Matrices Do?
Graphs
Sparsity

Norms and Errors
The 'Undo' Button for Linear Operations: LU

LU: Applications
Linear Algebra Applications
Interpolation

Repeating Linear Operations:
Eigenvalues and Steady States
Eigenvalues: Applications
Approximate Undo: SVD and Least Squares

SVD: Applications

Solving Funny-Shaped Linear Systems
Data Fitting
Norms and Condition Numbers
Low-Rank Approximation

Iteration and Convergence

Solving One Equation
Solving Many Equations
Finding the Best: Optimization in 1D
Optimization in n Dimensions