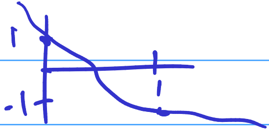
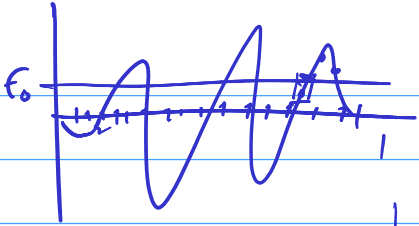
$$\kappa_{abs} = \max_{x\in[0,1]} \frac{\text{absolute error of output}}{\text{absolute error in input}}$$

$$= \max_{x\in[0,1]} \frac{|f(x+h) - f(x)|}{|h|}$$

$$\approx \max_{x\in[0,1]} |f'(x)|$$

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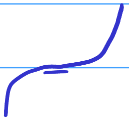

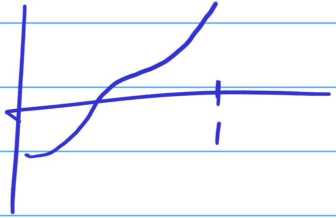
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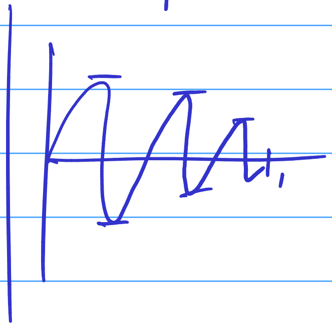
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Modelling the World with Matrices

Predicting Movie Popularity

vectors: $x^{(i)}$ - preferences of friend i

$$A = \begin{bmatrix} \text{movie 1 ads} \\ \text{movie 2 ads} \\ - \\ \vdots \end{bmatrix}$$

Goal:

$$y = Ax$$

$$y = \sum_{i=1}^{n} A x^{(i)}$$

$$Y = \begin{bmatrix} \quad \end{bmatrix} = A \cdot \begin{bmatrix} x^{(1)} & x^{(2)} & \dots & x^{(n)} \end{bmatrix}$$

$Y = A \cdot X$ Part 1: given A, X

$y_i = \sum_{j=1}^{\hat{}} Y_{ij}$ Part 2: given A, Y

A_{ij} = jth attribute of the ith movie

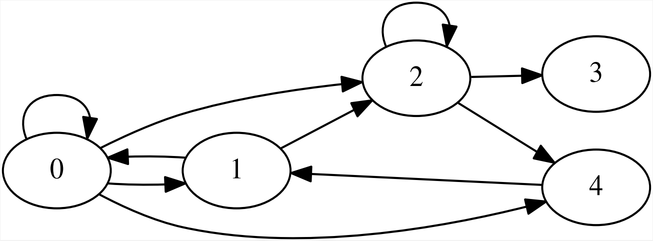
X_{jk} = preference of friend k with respect to attribute j

la. solve

Outline

Graphs as Matrices

How could this (directed) graph be written as a matrix?



$$A \underset{\#\text{vertices}}{=} \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \qquad A_{ji} = 1 \text{ if } \exists\, (i,j) \in E$$

$$G = (V, E)$$

Matrices for Graph Traversal: Technicalities

What is the general rule for turning a graph into a matrix?

$A_{ji} = 1$ if there is an edge from i to j

What does the matrix for an *undirected* graph look like?

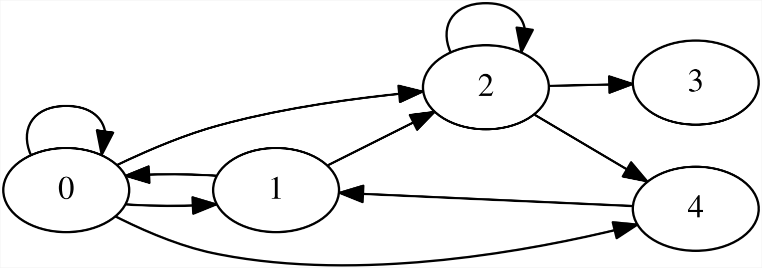
symmetric

How could we turn a *weighted graph* (i.e. one where the edges have weights–maybe 'pipe widths') into a matrix?

A_{ji} is the weight of edge $i \to j$

Graph Matrices and Matrix-Vector Multiplication

If we multiply a graph matrix by the ith unit vector, what happens?



$$A \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \, ? \, = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

Demo: Matrices for Graph Traversal

Outline

Storing Sparse Matrices

Some types of matrices (including graph matrices) contain many zeros.
Storing all those zero entries is wasteful.
How can we store them so that we avoid storing tons of zeros?

Storing Sparse Matrices Using Arrays

How can we store a sparse matrix using just arrays? For example:

$$\begin{pmatrix} 0 & 2 & 0 & 3 \\ 1 & 4 & & \\ & & 5 & \\ 6 & & & 7 \end{pmatrix}$$