

Overview

Graphs as mat

HW 5 ad

Sparse matrices

Exempel 3 dates

Norms

$\{0, 1\}^n$

$Q_{13, 5}$

$$\{ \overset{\downarrow}{0, 1} \} \times \{ \underline{3, 4, 5} \} \times \{ \underline{7, 8, 9} \}$$

$$\begin{pmatrix} 0 \\ 4 \\ 7 \end{pmatrix}$$

$$\{0, 1\}^n = \{0, 1\} \times \{0, 1\} \times \{0, 1\} \dots$$

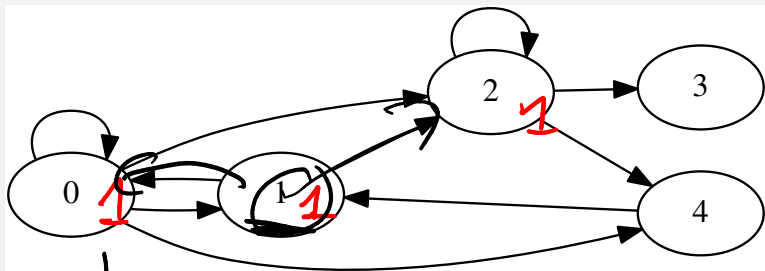
$$\{0, 1\}^n \subseteq [-1, 1]^n$$

$$\begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ \vdots \end{pmatrix}$$

$$\begin{pmatrix} -0.9 \\ 0.5 \\ \vdots \\ 0.3 \end{pmatrix}$$

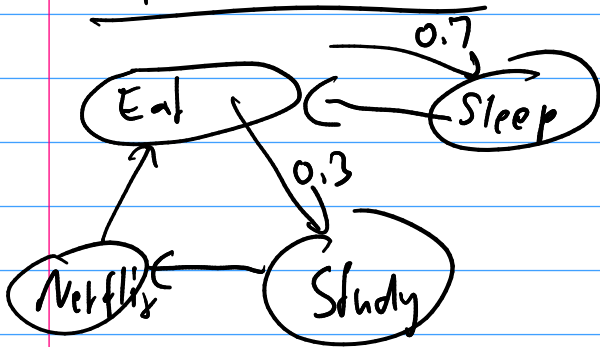
Graph Matrices and Matrix-Vector Multiplication

If we multiply a graph matrix by the i th unit vector, what happens?



$$\begin{matrix} 0 \rightarrow \\ 1 \rightarrow \\ 2 \rightarrow \\ 3 \rightarrow \\ \rightarrow \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Markov chain



Markov property: Only consider current state for $p(\text{next state})$.

$$\begin{array}{l}
 \text{Eat} \rightarrow \\
 \text{Sleep} \rightarrow \\
 \text{Nxt} \rightarrow \\
 \text{Study} \rightarrow
 \end{array}
 \begin{array}{c}
 \downarrow^E \\
 \downarrow^S \\
 \downarrow^N \\
 \downarrow^{\text{Study}}
 \end{array}
 \begin{pmatrix}
 0 & 1 & 1 & 0 \\
 0.7 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 \\
 0.3 & 0 & 0 & 0
 \end{pmatrix}
 =
 \begin{pmatrix}
 1 \\
 0 \\
 0 \\
 0
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 0.7 \\
 0 \\
 .3
 \end{pmatrix}$$

Demo: Matrices for Graph Traversal

Outline

Python, Numpy, and Matplotlib
Making Models with Polynomials
Making Models with Monte Carlo

Error, Accuracy and Convergence
Floating Point

Modeling the World with Arrays

The World in a Vector

What can Matrices Do?

Graphs

Sparsity

Norms and Errors
The 'Undo' Button for Linear Operations: LU

LU: Applications

Linear Algebra Applications

Interpolation

Repeating Linear Operations:
Eigenvalues and Steady States

Eigenvalues: Applications

Approximate Undo: SVD and Least Squares

SVD: Applications

Solving Funny-Shaped Linear Systems

Data Fitting

Norms and Condition

Numbers

Low-Rank Approximation

Iteration and Convergence

Solving One Equation

Solving Many Equations

Finding the Best: Optimization in 1D

Optimization in n Dimensions

Storing Sparse Matrices

Some types of matrices (including graph matrices) contain many zeros.

Storing all those zero entries is wasteful.

How can we store them so that we avoid storing tons of zeros?

$$\begin{pmatrix} 0 & 3 \\ 7 & 0 \end{pmatrix}$$

$$\{ 0 : \{ 1 : 3 \}, 1 : \{ 0 : 7 \} \}$$

Storing Sparse Matrices Using Arrays

How can we store a sparse matrix using just arrays? For example:

$$\begin{pmatrix} 0 & 2 & 0 & 3 \\ 1 & 4 & & \\ & & 5 & \\ 6 & & & 7 \end{pmatrix}$$

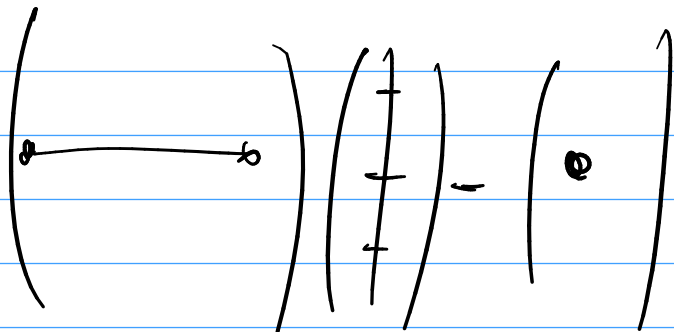
CSR

↑
Compressed
Sparse
Row
Row 3

Row Starts = (0, 2, 4, 5, 7)

Column Indices = (1, 3, 0, 2, 0, 3)

Values = (2, 3, 1, 4, 5, 6, 7)



Mat-vec: $O(nnz)$

Mat-mat: ?!

$$\begin{pmatrix} 3 & 4 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\text{Row Starts} = (0, 2, 2, 3)$$

$$\text{colIdx} = (0, 1, 0)$$

$$\text{values} = (3, 4, 1)$$

Demo: Sparse Matrices in CSR Format

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What's a norm?

Define **norm**.