Overview

Graphs as mat | HW5 due
Sparse matrices | Exam Dec 3 dates
Norms

$\{0,1\}^n$

Q1,3,5

$$\underline{\{0,1\}} \times \underline{\{3,4,5\}} \times \{7,8,9\}$$

$$\begin{pmatrix} 0 \\ 4 \\ 7 \end{pmatrix}$$

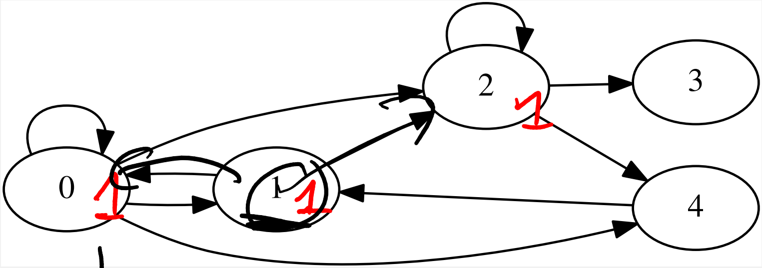
$$\{0,1\}^n = \{0,1\} \times \{0,1\} \times \{0,1\} \ldots$$

$$\{0,1\}^n \subseteq [-1,1]^n$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ \vdots \end{pmatrix} \qquad \begin{pmatrix} -0.8 \\ 0.5 \\ \vdots \\ 0.3 \end{pmatrix}$$

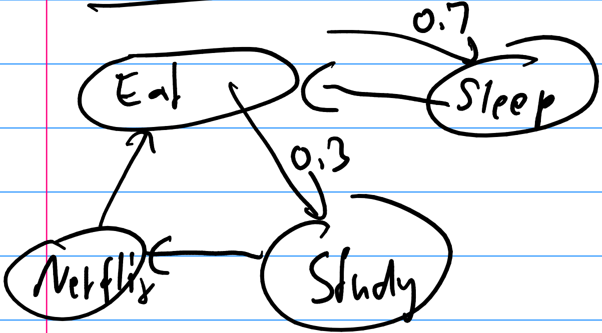
Graph Matrices and Matrix-Vector Multiplication

If we multiply a graph matrix by the ith unit vector, what happens?



$$\begin{array}{c} 0 \to \\ 1 \to \\ 2 \to \\ 3 \to \\ \to \end{array} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Markov chain



Markov property: Only consider current state for p(next state).

$$
\begin{array}{r c}
 & \begin{array}{cccc} E & S & N & \text{Study} \\ \downarrow & \downarrow & \downarrow & \downarrow \end{array} \\
\begin{array}{r} \text{Eat} \rightarrow \\ \text{Sleep} \rightarrow \\ \text{Netf} \rightarrow \\ \text{study} \rightarrow \end{array} &
\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.3 & 0 & 0 & 0 \end{pmatrix}
\end{array}
\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}
=
\begin{array}{c} \downarrow \\ \begin{pmatrix} 0 \\ 0.7 \\ 0 \\ .3 \end{pmatrix} \end{array}
$$

Demo: Matrices for Graph Traversal

Outline

Storing Sparse Matrices

Some types of matrices (including graph matrices) contain many zeros.
Storing all those zero entries is wasteful.
How can we store them so that we avoid storing tons of zeros?

$$\begin{pmatrix} 0 & 3 \\ 7 & 0 \end{pmatrix}$$

$$\{0 : \{1 : 3\}, 1 : \{0 : 7\}\}$$

Storing Sparse Matrices Using Arrays

How can we store a sparse matrix using just arrays? For example:

$$\begin{pmatrix} 0 & 2 & 0 & 3 \\ 1 & 4 & & \\ & & 5 & \\ 6 & & & 7 \end{pmatrix}$$

CSR

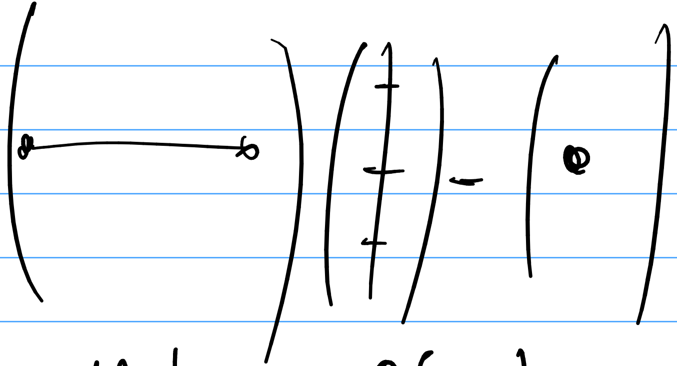
Compressed
Sparse
Row

Row 3

RowStarts = (0 2 4 5 , 7)

ColumnIndices = (1 3 0 1 2 0 3)

Values = (2 3 1 4 5 6 7)



Mat-vec: $O(\text{nnz})$

Mat-mat: ?!

$$\begin{pmatrix} 3 & 4 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Row Starts = (0, 2, 2, 3)

Col Idx = (0 1 0)

Values = (3 4 1)

Demo: Sparse Matrices in CSR Format

Outline

Norms

What's a norm?

Define norm.