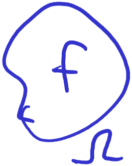
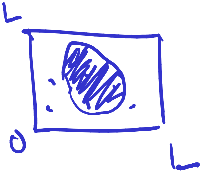
Monte Carlo Overview

$$E[Y] = \int y\,p(y)\,dy$$



$$G = \iiint_{\Omega} f(x,y,z)\,dx\,dy\,dz$$



$$G \approx \frac{1}{N}\sum_{i=1}^{N} f(s_i)\cdot p(s_i)$$

$$\int_0^L\int_0^L p(x)\,dx = 1$$

$$\delta(x,y) = 1 \quad \text{if} \quad (x,y) \in \Omega$$
$$= 0 \quad \text{otherwise}$$

$$G = \int_0^L \int_0^L f(x,y) \cdot \delta(x,y)\, dx\, dy$$
$$= \iint_\Omega f(x,y)\, dx\, dy$$

$$p(x,y) = \underline{\delta(x,y) / |\Omega|}$$

$$G = E[f(X,Y)]$$

where E is distributed according to $p(x,y)$

$$G \approx \frac{1}{N} \sum_{i=1}^{n} f(s_i)$$

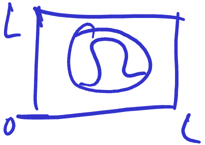
if s_i is in Ω

$$G \approx \frac{1}{N} \frac{\Omega}{L^2} \sum_{i=1}^{n} f(s_i)\,\delta(s_i)$$

$$G \approx \frac{1}{N_{in}} \sum_{i=1}^{N} f(s_i)\, \delta(s_i)$$

$$N_{in} = \sum_{i=1}^{N} \delta(s_i)$$

$$|\Omega| = \int_{\Omega} 1\, dx$$



$$|\Omega| \approx \frac{1}{N} \cdot L^2 \cdot \sum_{i=1}^{N} f(s_i)$$

$f(s_i) = \frac{1}{|\Omega|}$ when $s_i \in \Omega$

f to be 1 inside Ω

$$N_{in} = \sum_{i=1}^{N} f(s_i)$$

$$G = \iint_{\Omega} h(x,y)\,dx\,dy$$

$$= \int_0^L \int_0^2 h(x,y)\cdot\delta(x,y)\,dx\,dy$$

$\delta(x,y)$ is 1 inside Ω

define probability distribution $p(x,y)$ so that $1 = \int_0^L \int_0^L p(x,y)\,dx\,dy$

$$p(x,y) = \frac{1}{|\Omega|}\delta(x,y)$$

$$E[h(x,y)] = G/|\Omega|$$

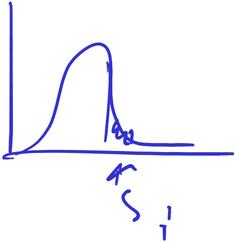
$$E[h(x,y)] = \int_0^1\int_0^1 h(x,y)\,p(x,y)\,dx\,dy$$

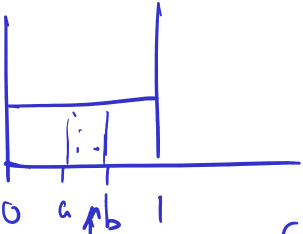
$$= \int_0^1\int_0^1 \frac{1}{|\Omega|}\,\delta(x,y)\,h(x,y)\,dx\,dy$$

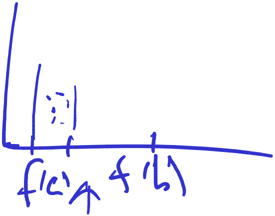
$$\approx \frac{1}{N}\sum_{i=1}^{N} h(s_i)$$

s_i is sampled according to $p(x,y)$

$p(x)$







$$s_i \longrightarrow f \longrightarrow r_i$$

$$r_i = f(s_i)$$

$$S_N = X_1 + \dots + X_N$$

$$E\left[\frac{S_N}{N} - E[X]\right]$$

$$E\left[\left(\frac{S_N}{N}\right)^2 - E[X]^2\right]$$

$$E\left[\left(\frac{S_N}{N}\right)^2\right]$$