

Overview

Announcements

HW 4

Examlet 2

Vectors

What's a vector?

Vectors in the 'Real World'

Demo: Images as Vectors

Demo: Sound as Vectors

Demo: Shapes as Vectors ✓

Outline

Python, Numpy, and Matplotlib
Making Models with Polynomials
Making Models with Monte Carlo

Error, Accuracy and Convergence
Floating Point

Modeling the World with Arrays

The World in a Vector
What can Matrices Do?
Graphs
Sparsity

Norms and Errors
The 'Undo' Button for Linear Operations: LU

LU: Applications
Linear Algebra Applications
Interpolation

Repeating Linear Operations:
Eigenvalues and Steady States
Eigenvalues: Applications
Approximate Undo: SVD and Least Squares

SVD: Applications

Solving Funny-Shaped Linear Systems
Data Fitting
Norms and Condition Numbers
Low-Rank Approximation

Iteration and Convergence

Solving One Equation
Solving Many Equations
Finding the Best: Optimization in 1D
Optimization in n Dimensions

Matrices

What does a matrix do?

It represents a *linear function* between two vector spaces $f : U \rightarrow V$ in terms of bases $\mathbf{u}_1, \dots, \mathbf{u}_n$ of U and $\mathbf{v}_1, \dots, \mathbf{v}_m$ of V . Let

$$\mathbf{u} = \alpha_1 \mathbf{u}_1 + \dots + \alpha_n \mathbf{u}_n$$

and

$$\mathbf{v} = \beta_1 \mathbf{v}_1 + \dots + \beta_m \mathbf{v}_m.$$

Then f can *always* be represented as a matrix that obtains the β s from the α s:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix}.$$

Example: The 'Frequency Shift' Matrix

Assume both \mathbf{u} and \mathbf{v} are linear combination of sounds of different frequencies:

$$\mathbf{u} = \alpha_1 \mathbf{u}_{110 \text{ Hz}} + \alpha_2 \mathbf{u}_{220 \text{ Hz}} + \cdots + \alpha_4 \mathbf{u}_{880 \text{ Hz}}$$

(analogously for \mathbf{v} , but with β s). What matrix realizes a 'frequency doubling' of a signal represented this way?

Matrices in the 'Real World'

What are some examples of matrices in applications?

Demo: Matrices for Geometry Transformation

Demo: Matrices for Image Blurring

In-class activity: Computational Linear Algebra

Outline

Python, Numpy, and Matplotlib
Making Models with Polynomials
Making Models with Monte Carlo

Error, Accuracy and Convergence
Floating Point

Modeling the World with Arrays

The World in a Vector

What can Matrices Do?

Graphs

Sparsity

Norms and Errors
The 'Undo' Button for Linear Operations: LU

LU: Applications

Linear Algebra Applications

Interpolation

Repeating Linear Operations:
Eigenvalues and Steady States

Eigenvalues: Applications

Approximate Undo: SVD and Least Squares

SVD: Applications

Solving Funny-Shaped Linear Systems

Data Fitting

Norms and Condition

Numbers

Low-Rank Approximation

Iteration and Convergence

Solving One Equation

Solving Many Equations

Finding the Best: Optimization in 1D

Optimization in n Dimensions

Graphs as Matrices

How could this (directed) graph be written as a matrix?

