

Overview

- Equation in \mathbb{R}^n
in \mathbb{R}^n
- Optimierung

$$\underbrace{f(x) = y}_{\text{circled}} \Leftrightarrow g(x) = 0$$

$$g(x) = f(x) - y$$

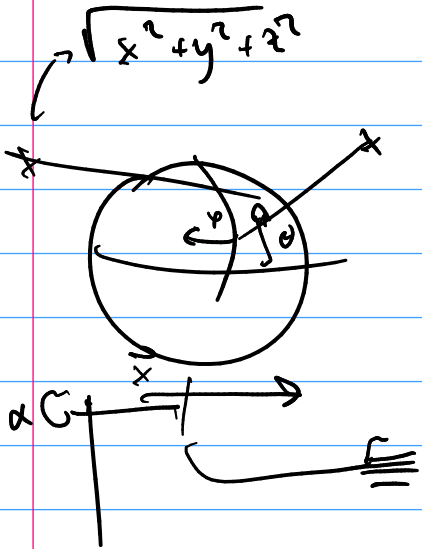
Applications:

$$\rightarrow \sqrt{y} = x \Leftrightarrow y = x^2$$

$$\cdot \frac{1}{5} = x$$

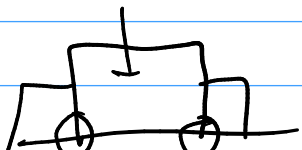
$$\Leftrightarrow 5x - 1 = 0$$

\uparrow
Newton's method



$$f(\vec{v}, \theta) = \begin{pmatrix} \text{disk 1} \\ \vdots \\ \text{disk } n \end{pmatrix}$$

Trash

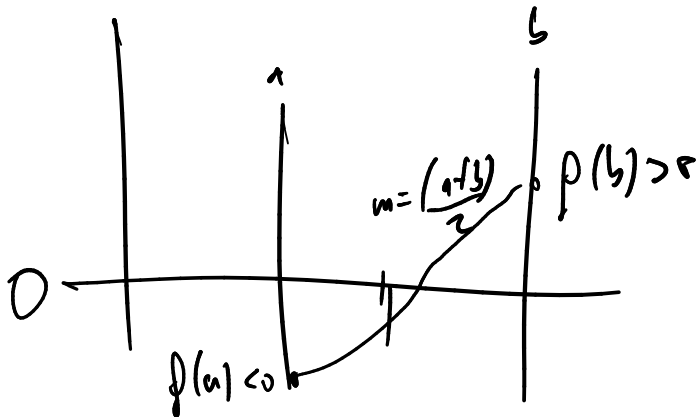


$$\mathcal{P} \begin{pmatrix} \text{joint angle} \\ \text{joint angle} \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

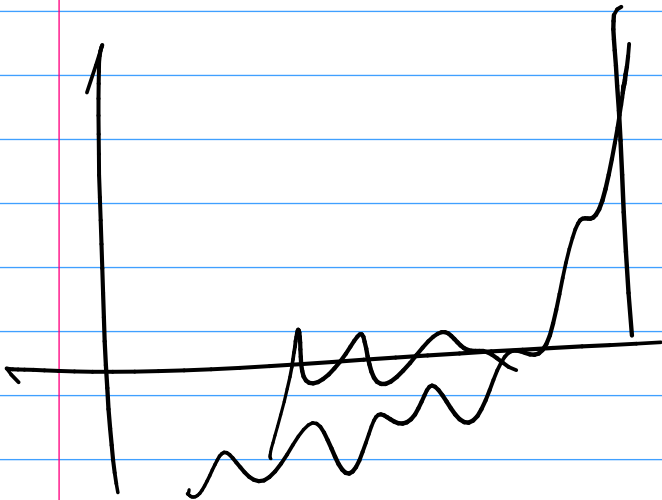
Solving Nonlinear Equations

What is the goal here?

find x so that $f(x) = 0$



$$e_{k+1} = C \cdot e_k \quad \left| \quad e_k = b_k^{-1} a_n$$



Bisection Method

Assume continuous function f has a zero on the interval $[a, b]$ and

$$\text{sign}(f(a)) = -\text{sign}(f(b)).$$

Perform binary search: check sign of $f((a + b)/2)$ and define new search interval so that ends have opposite sign.

Demo: Bisection Method

What's the rate of convergence? What's the constant?

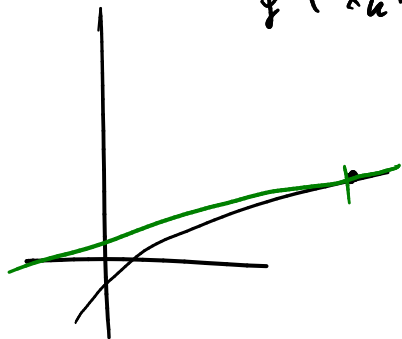
Newton's Method

Derive Newton's method.

$$f(x) = 0$$

$$x_k + h$$

$$\tilde{f}(x_k + h) = f(x_k) + h \cdot f'(x_k)$$



$$f(x_k) + h \cdot f'(x_k) = 0$$

$$h \cdot f'(x_k) = -f(x_k)$$

$$h = -\frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k + h = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$e_{k+1} \leq C \cdot e_k^2$$

Demo: Newton's method

Demo: Convergence of Newton's Method

What are some **drawbacks** of Newton?

Secant Method

What would Newton without the use of the derivative look like?

Demo: Secant Method

In-class activity: Nonlinear equations in 1D

Outline

Python, Numpy, and Matplotlib
Making Models with Polynomials
Making Models with Monte Carlo
Error, Accuracy and Convergence
Floating Point
Modeling the World with Arrays
 The World in a Vector
 What can Matrices Do?
 Graphs
 Sparsity
Norms and Errors
The 'Undo' Button for Linear Operations: LU
Repeating Linear Operations:
Eigenvalues and Steady States
Eigenvalues: Applications

Approximate Undo: SVD and Least Squares
SVD: Applications
 Solving Funny-Shaped Linear Systems
 Data Fitting
 Norms and Condition Numbers
 Low-Rank Approximation
Interpolation
Iteration and Convergence
Solving One Equation
Solving Many Equations
Finding the Best: Optimization in 1D
Optimization in n Dimensions