Overview

- Equation in 1D
  in nD
- Optimization

$$f(x) = y \iff g(x) = 0$$

$$g(x) = f(x) - y$$

Applications:

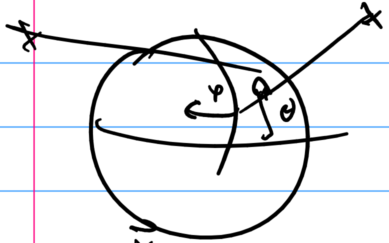
$\rightarrow$ $\sqrt{y} = x \iff y = x^2$

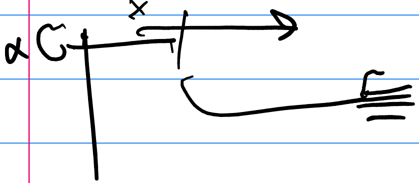
Newton's method

- $\frac{1}{5} = x$

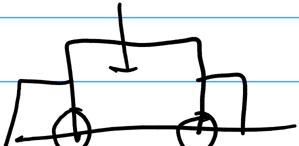
$\iff 5x - 1 = 0$

$\sqrt{x^2+y^2+z^2}$





$$f(\varphi, \theta) = \begin{pmatrix} dist_1 \\ \vdots \\ dist_n \end{pmatrix}$$

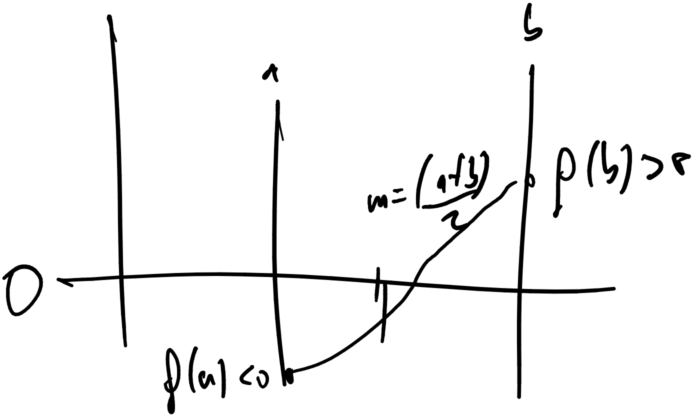


$$f\begin{pmatrix} \text{joint angle 1} \\ \vdots \\ \text{joint angle n} \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

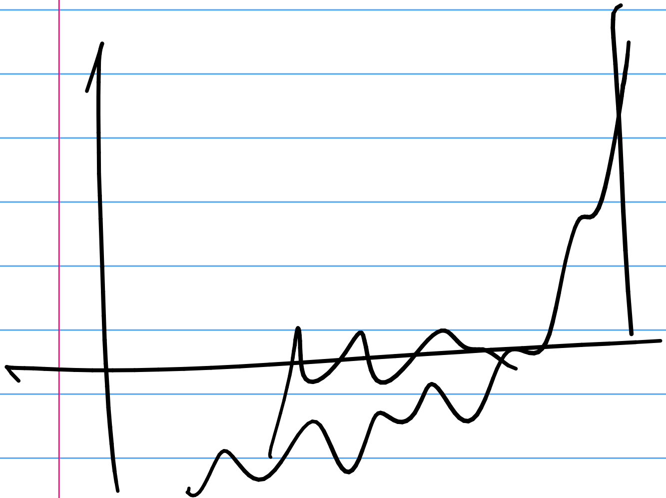
Solving Nonlinear Equations

What is the goal here?

find x so that $f(x)=0$



$$e_{k+1} = C \cdot e_k \quad \Big| \; e_k = b_k - a_n$$



Bisection Method

Assume continuos function f has a zero on the interval $[a, b]$ and

$$\text{sign}(f(a)) = -\text{sign}(f(b)).$$

Perform binary search: check sign of $f((a+b)/2)$ and define new search interval so that ends have opposite sign.

Demo: Bisection Method

> What's the rate of convergence? What's the constant?

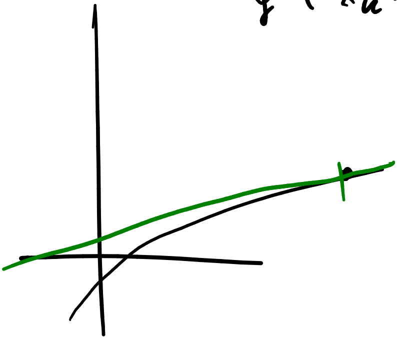
Newton's Method

Derive Newton's method.

$f(x) = 0$

$x_k + h$

$\tilde{f}(x_k + h) = f(x_k) + h \cdot f'(x_k)$



$f(x_k) + h \cdot f'(x_k) = 0$

$h \cdot f'(x_k) = -f(x_k)$

$h = -\frac{f(x_k)}{f'(x_k)}$

$$x_{k+1} = x_k + h = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$e_{k+1} \leq C \cdot e_k^2$$

Demo: Newton's method
Demo: Convergence of Newton's Method

What are some **drawbacks** of Newton?

Secant Method

What would Newton without the use of the derivative look like?

Demo: Secant Method

In-class activity: Nonlinear equations in 1D

Outline