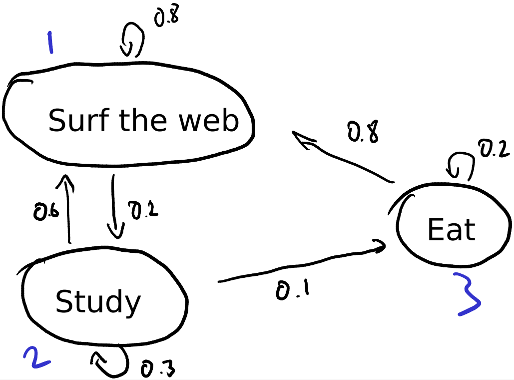
Outline

Eigenvalue problems apps
- steady state Markov
- time-dependent

SVD
- nonsymmetric eigenvalue decomp
- least-squares, reduced representation

Markov chains and Eigenvalue Problems

Recall our example of a Markov chain:



Suppose this is an accurate model of the behavior of the average student. :) How likely are we to find the average student in each of these states?

Demo: Finding an equilibrium distribution using the power method

steady state distribution

$$A = \begin{bmatrix} .8 & .6 & .8 \\ .2 & .3 & 0 \\ 0 & .1 & .2 \end{bmatrix} \quad v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

A_{ij} is the prob. of transitioning from state j to i

$v = Av$

columns sum to 1

$\lambda_1 = 1 \longrightarrow$ largest eig-value

Understanding Time Behavior

Many important systems in nature are modeled by describing the time rate of change of something.

- E.g. every bird will have 0.2 baby birds on average per year.
- But there are also foxes that eat birds. Every fox present decreases the bird population by 1 birds a year. Meanwhile, each fox has 0.3 fox babies a year. And for each bird present, the population of foxes grows by 0.9 foxes.

Set this up as equations and see if eigenvalues can help us understand how these populations will evolve over time.

$$\underbrace{\begin{array}{c c} \text{birds} & \\ \text{foxes} & \end{array}\begin{bmatrix} .2 & -1 \\ .9 & .3 \end{bmatrix}}_{A}$$

year 0 p_0

year 1 $p_1 = A \cdot p_0$

b - birds   f - foxes

$$\frac{\partial}{\partial t} b = .2b - 1.f$$

$$\frac{\partial}{\partial t} f = .9b + .3f$$

$$p = \begin{pmatrix} b \\ f \end{pmatrix}, \quad \underline{\frac{dp}{dt} = A \cdot p}$$

What is $p(t)$?

$$p(t) = e^{\lambda t} p_0$$

$$\frac{\partial p}{\partial t} = \lambda p(t)$$

$$\lambda p_0 = A p_0$$