\[ x \in (0, 1) \]
\[ f(1) = 1 \]
\[ \frac{5e^{-10}}{\text{max}} = \text{min} \]
\[ 1 + (x + y - 4/3) \]

Absoulute error of all pol
Modelling the World with Matrices

Predicting Movie Popularity

Vectors: \( x \) - preferences of friend \( i \)

Goal:

\[
\begin{align*}
A &= \begin{bmatrix}
mov.1 & \cdots & mov.\, n \end{bmatrix} \\

\hat{y} &= \sum_{i=1}^{n} Ax^{(i)}
\end{align*}
\]
\[ Y = A \cdot X \]

**Part 1:** given \( A, X \)

\[ y_i = \sum_{j=1}^{n} Y_{ij} \]

**Part 2:** given \( A, Y \)

\[ A_{ij} = j \text{th attribute of the } i \text{th movie} \]

\[ X_{jk} = \text{preference of movie } k \text{ with respect to attribute } j \]

la. solve
Outline

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Making Models with Polynomials
Making Models with Monte Carlo
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Floating Point
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  The World in a Vector
  What can Matrices Do?
  Graphs
  Sparsity
Norms and Errors
The ‘Undo’ Button for Linear Operations: LU
LU: Applications
  Linear Algebra Applications
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Repeating Linear Operations: Eigenvalues and Steady States
Eigenvalues: Applications
Approximate Undo: SVD and Least Squares
SVD: Applications
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Solving One Equation
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Finding the Best: Optimization in 1D
Optimization in $n$ Dimensions
Graphs as Matrices

How could this (directed) graph be written as a matrix?

\[ A = \begin{bmatrix}
1 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
\end{bmatrix}^T \]

\[ A_{ij} = 1 \text{ if } \exists (i,j) \in E \]

\[ G = (V, E) \]
Matrices for Graph Traversal: Technicalities

What is the general rule for turning a graph into a matrix?

\[ A_{ji} = 1 \text{ if there is an edge from } i \text{ to } j \]

What does the matrix for an undirected graph look like?

Symmetric

How could we turn a weighted graph (i.e. one where the edges have weights—maybe ‘pipe widths’) into a matrix?

\[ A_{ji} \text{ is the weight of edge } i \rightarrow j \]
If we multiply a graph matrix by the $i$th unit vector, what happens?

$$A \left( \begin{array} {c} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right) = ? \times \left( \begin{array} {c} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right)$$
Demo: Matrices for Graph Traversal
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Storing Sparse Matrices

Some types of matrices (including graph matrices) contain many zeros.
Storing all those zero entries is wasteful.
How can we store them so that we avoid storing tons of zeros?
How can we store a sparse matrix using just arrays? For example:

$$
\begin{pmatrix}
0 & 2 & 0 & 3 \\
1 & 4 & & 5 \\
6 & & 7 \\
\end{pmatrix}
$$