Overview

Graphs as matrices

Sparse matrices

Noms

HWS 3

Excerpt of 3 dates

Q1, 19

Q17, 5
$\{0,1\} \times \{3,4,5\} \times \{7,8,9\}$

\[
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
\]

$\{0,1\}^n = \{0,1\} \times \{0,1\} \times \{0,1\} \ldots$
$\mathcal{S}_{Q_1}^n \subseteq [-1, 1]^n$

\[
\begin{pmatrix}
0 \\
1 \\
0 \\
1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
-0.8 \\
0.5 \\
| \\
0.2
\end{pmatrix}
\]
If we multiply a graph matrix by the $i$th unit vector, what happens?
Markov chain

Markov property: Only consider the current state for $p$ (next state).
Demo: Matrices for Graph Traversal
Some types of matrices (including graph matrices) contain many zeros. Storing all those zero entries is wasteful. How can we store them so that we avoid storing tons of zeros?
How can we store a sparse matrix using just arrays? For example:

\[
\begin{pmatrix}
0 & 2 & 0 & 3 \\
1 & 4 & \ & 5 \\
6 & & 7
\end{pmatrix}
\]

Row Starts = [0, 2, 4, 5, 7]

Column Indices = (1 3 0 1 2 0 3)

Values = [2 3 1 4 5 6 7]
\[
\begin{pmatrix}
\vdots \\
0
\end{pmatrix}
\begin{pmatrix}
\vdots \\
\vdots \\
0
\end{pmatrix}
= 
\begin{pmatrix}
0
\end{pmatrix}
\]

\text{Mat-vec: } O(n^2)

\text{Mat-mat: ?!}
\[
\begin{pmatrix}
3 & 4 \\
0 & 0 \\
1 & 0
\end{pmatrix}
\]

Row Statis = \((0, 2, 2, 3)\)

Col \(\text{Idx}\) = \((0, 1, 0)\)

Values = \((3, 4, 1)\)
Demo: Sparse Matrices in CSR Format
Outline

Python, Numpy, and Matplotlib
Making Models with Polynomials
Making Models with Monte Carlo
Error, Accuracy and Convergence
Floating Point
Modeling the World with Arrays
  The World in a Vector
  What can Matrices Do?
  Graphs
  Sparsity

Norms and Errors
The ‘Undo’ Button for Linear Operations: LU
LU: Applications
  Linear Algebra Applications
  Interpolation

Repeating Linear Operations:
  Eigenvalues and Steady States
Eigenvalues: Applications
Approximate Undo: SVD and Least Squares
SVD: Applications
  Solving Funny-Shaped Linear Systems
  Data Fitting
  Norms and Condition Numbers
  Low-Rank Approximation
Iteration and Convergence
Solving One Equation
Solving Many Equations
Finding the Best: Optimization in 1D
Optimization in $n$ Dimensions
Norms

What’s a norm?

Define norm.