Why polynomials?

\[ a_3 x^3 + a_2 x^2 + a_1 x + a_0 \]

- How do we write a general degree \( n \) polynomial?

\[ \sum_{i=0}^{n} a_i x^i = a_0 x^0 + a_1 x^1 + \cdots + a_n x^n \]

- Why polynomials and not something else?
Reconstructing a Function From Derivatives

- If we know \( f(x_0), f'(x_0), f''(x_0) \), can we approximately reconstruct the function as a polynomial \( p \)?

\[
p(x) = ??? + ???x + ???x^2 + \ldots
\]

\[
f(0), f'(0), f''(0)
\]

\[
p(x) = a + bx + cx^2 + \ldots
\]

\[
p(0) = a
\]

\[
p'(x) = b + 2cx + 3dx^2
\]

\[
p'(0) = b
\]

\[
p''(x) = 2c + 6dx
\]

\[
p''(0) = 2c
\]

\[
c = p''(0)/2
\]


\[
g(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} x^i
\]

\[
g(x) = f(x - x_0)
\]

**Demo:** Polynomial Approximation with Derivatives (Part I)

\[
f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(x_0)}{i!} (x - x_0)^i
\]
Shifting the Expansion Center

○ Can you do this at points other than the origin?

\[ g(x) = f(x + x_0) \]