Objectives

1. Design a numerical experiment in Python+numpy+scipy
2. Write efficient Python+numpy by avoiding loops
3. Study how arrays of data scale in Python
4. Observe the limits of precision
1. An experiment

- Brownian motion

Brownian motion is the random motion of particles suspended in a fluid (a liquid or a gas) resulting from their collision with the quick atoms or molecules in the gas or liquid. The term "Brownian motion" can also refer to the mathematical model used to describe such random movements, which is often called a particle theory. -Wikipedia

\[ x(t + \Delta t) = x(t) + \mathcal{N}(0, \sigma^2 \Delta t; t, t + \Delta t) \]

- update
- random amount

- particle position
1. An experiment

- Brownian motion

\[ x(t + \Delta t) = x(t) + \mathcal{N}(0, \sigma^2 \Delta t; t, t + \Delta t) \]
1. An experiment

- Write an algorithm!
- How does the distance from the origin depend on $n$, the number of steps?
- Is our algorithm efficient?
- Can we repeat the algorithm?
2. Loops

- Is Python slow?
  - yes: interpreted, dynamically typed
  - no: can call libraries and JIT
2. Loops

- https://jakevdp.github.io/blog/2014/05/09/why-python-is-slow/

- Dynamic typing
2. Loops

- https://jakevdp.github.io/blog/2014/05/09/why-python-is-slow/

```c
/* C code */
int a = 1;
int b = 2;
int c = a + b;
```

1. Assign `<int>` 1 to `a`
2. Assign `<int>` 2 to `b`
3. call `binary_add<int, int>(a, b)`
4. Assign the result to `c`
2. Loops

- https://jakevdp.github.io/blog/2014/05/09/why-python-is-slow/

```python
# python code
a = 1
b = 2
c = a + b
```

1. Assign 1 to a
   - **1a.** Set `a` to `PyObject_HEAD->typecode` to `integer`
   - **1b.** Set `a` to `val = 1`

2. Assign 2 to b
   - **2a.** Set `b` to `PyObject_HEAD->typecode` to `integer`
   - **2b.** Set `b` to `val = 2`

3. call `binary_add(a, b)`
   - **3a.** find `typecode` in `a` to `PyObject_HEAD`
   - **3b.** `a` is an integer; value is `a` to `val`
   - **3c.** find `typecode` in `b` to `PyObject_HEAD`
   - **3d.** `b` is an integer; value is `b` to `val`
   - **3e.** call `binary_add<int, int>(a, b)`
   - **3f.** result of this is `result`, and is an integer.

4. Create a Python object `c`
   - **4a.** set `c` to `PyObject_HEAD->typecode` to `integer`
   - **4b.** set `c` to `val` to `result`
2. Loops

- https://jakevdp.github.io/blog/2014/05/09/why-python-is-slow/

- Compilers optimize during compilation process
  
  - repeated operations

  - caching

  - etc

- Interpreted codes need JIT or C extensions. This is possible, just an extra step in Python
2. Loops

- [https://jakevdp.github.io/blog/2014/05/09/why-python-is-slow/](https://jakevdp.github.io/blog/2014/05/09/why-python-is-slow/)
- Lists have similar advantages/disadvantages.
  - In C, you would use a buffered array
  - With bumpy you get this
2. Loops

- Let’s look at an example: average the pixels
2. Loops

• Let’s look at an example: average the pixels

\[
pixel \leftarrow \frac{1}{8} (4 \cdot \pixel + \pixel_{north} + \pixel_{south} + \pixel_{west} + \pixel_{east})
\]
3. How “fast” are arrays?

1D

\[ v = [v_0, v_1, \ldots, v_{n-1}] \]

\[ w = [w_0, w_1, \ldots, w_{n-1}] \]

2D

\[
A = \begin{bmatrix}
a_{00} & a_{01} & \cdots \\
\vdots & \vdots & \ddots \\
& & & a_{n-1,n-1}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
b_{00} & b_{01} & \cdots \\
\vdots & \vdots & \ddots \\
& & & b_{n-1,n-1}
\end{bmatrix}
\]
4. Limits of Precision

• What if we have a function and want to estimate its derivative?

• A simple test, but useful! We wouldn’t need to know the explicit derivative.

\[ f(x) \quad f'(x) \quad ? \]