Rayleigh Quotient Iteration

$A = X^{-1} D X$

Describe *Rayleigh Quotient Iteration*.

$$\frac{x^T A x}{x^T x}$$

Demo: Power Iteration and its Variants [cleared]

$(A^{-10})b$

A^{10}

$A = U\Sigma V^T$

$A^{-1} = V\Sigma^{-1}U^T$

$Ax = b$

$U\Sigma V^T x = b$

$\Sigma \underbrace{V^T x}_{y} = U^T b$

$y = V^T x$

$Vy = x$

$A^{10}x$

$AAA(\ldots(Ax))$

$= ((AAA)\quad)x \leftarrow$

$A \times x \; 10$

$A \times x\,(-10)$

$A * A$

In-Class Activity: Eigenvalues

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Schur form

with real eigenvalues and is non-defective

Show: Every matrix is orthonormally similar to an upper triangular matrix, i.e. $A = QUQ^T$. This is called the Schur form or Schur factorization.

$$Av = \lambda v \qquad V = \text{span}(\vec{v})$$

$$A : V \to V$$

$$: V^\perp \to V \oplus V^\perp$$

$$A = \underbrace{\begin{pmatrix} | & \text{Basis of} \\ v & \\ | & V^\perp \end{pmatrix}}_{Q_1} \begin{bmatrix} \lambda & \\ 0 & \boxed{} \\ \vdots & \end{bmatrix} \begin{pmatrix} Q_1^T \end{pmatrix}$$

Schur Form: Comments, Eigenvalues, Eigenvectors

$A = QUQ^T$. For complex λ:

- Either complex matrices, or
- 2×2 blocks on diag.

If we had a Schur form of A, how can we find the eigenvalues?

And the eigenvectors?