- Exam 2 - Vole or hw poll - Follow - ypsi complex starting vec on real valued untix 1 a+15 = 1 a-15 $- \hat{x}_i \qquad A \hat{v}_i = \lambda_i \hat{v}_i^2$ V= (v, ... v,) ₹3= E (8); V; 0, - V-1 x1 $\|\hat{x} - \hat{v_i}\| = h \Rightarrow |\mathcal{R}Q(x) - \lambda_i| = O(h^2)$ Symm. | - √ RQ(x) = 0 x is elyvect for Ini-> with

Goods

- Schur Form

- methods?

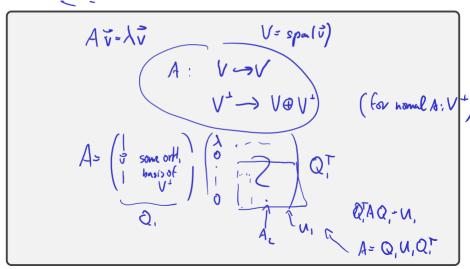
- orth. it.

~ Krylav

$$A = \times D \times^{-1}$$
 doesn't alongs exist \times^{-1} band if poorly condition

Schur form

Show: Every matrix is orthonormally similar to an upper triangular matrix, i.e. $A = QUQ^T$. This is called the Schur form or Schur factorization.



$$X = \alpha \vec{J} + \vec{d}_1 \vec{V}_2 + \cdots + \vec{d}_n \vec{V}_n$$

$$\vec{d} = \begin{pmatrix} \vec{d} \\ \alpha_1 \\ \alpha_n \end{pmatrix} = \vec{V}^{-1} \vec{X} = \vec{V}^{-2} \vec{X}$$

Do that recursively, on Az A- Q, AL QT 4, Q1 1 Ar Q OT E Schur form. Ceiganalues on diagonal

Schur Form: Comments, Eigenvalues, Eigenvectors

- Fig. $A = QUQ^T$. For complex λ :

 Either complex matrices, or

 2×2 blocks on diag. \longrightarrow with real valued Schur form

If we had a Schur form of A (no 2×2 blocks), can we find the eigenvalues?

And the eigenvectors?

Schur Form: Comments, Eigenvalues, Eigenvectors

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- ► Either complex matrices, or
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Computing Multiple Eigenvalues

All Power Iteration Methods compute one eigenvalue at a time. What if I want *all* eigenvalues?

Simultaneous Iteration

What happens if we carry out power iteration on multiple vectors simultaneously?

Orthogonal Iteration

$$X_0 = \text{southly wonder}$$

$$X_{i+1} = A \times_i$$

$$Q_0 = X_{i+1}$$

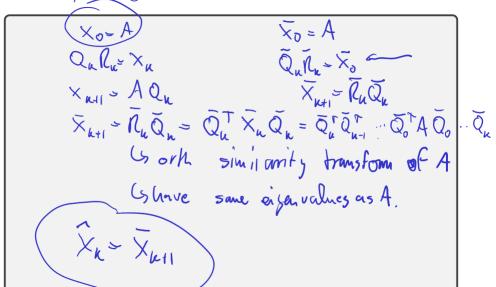
$$\times_i = Q_0$$

Toward the QR Algorithm

Demo: Orthogonal Iteration [cleared]

conveyed to Schur form

QR Iteration/QR Algorithm



Proof sketch: Equivalence of QR iteration/Orth. iteration

Orthogonal Iteration (no bars)

- $X_0 := A$
 - $ightharpoonup Q_0 R_0 := X_0,$
 - where we may choose $Q_0 = \bar{Q}_0$
 - $\hat{X}_0 = Q_0^H A Q_0 = Q_0^H Q_0 R_0 Q_0 = R_0 Q_0$
- $X_1 := AQ_0$
 - ▶ $Q_1R_1 := X_1$, and because of $X_1 = Q_0Q_0^HAQ_0 = Q_0\bar{X}_1 =$ $Q_0\bar{Q}_1\bar{R}_1$ we may choose $Q_1 = Q_0\bar{Q}_1 = \bar{Q}_0\bar{Q}_1$.

•

QR Iteration (with bars)

- $\bar{X}_0 := A$ $\bar{Q}_0 \bar{R}_0 := A$
- $ar{X}_1 := ar{R}_0 ar{Q}_0 = \hat{X}_0$ $ar{Q}_1 ar{R}_1 := ar{X}_1$
- $ar{X}_2 := ar{R}_1 ar{Q}_1$ $ar{X}_2 := Q_1^H A Q_1 = \hat{X}_1$

