bit.ly/cs450-f24

September 3, 2024

Announcements

- Quiz 3 glitchey grade remedy
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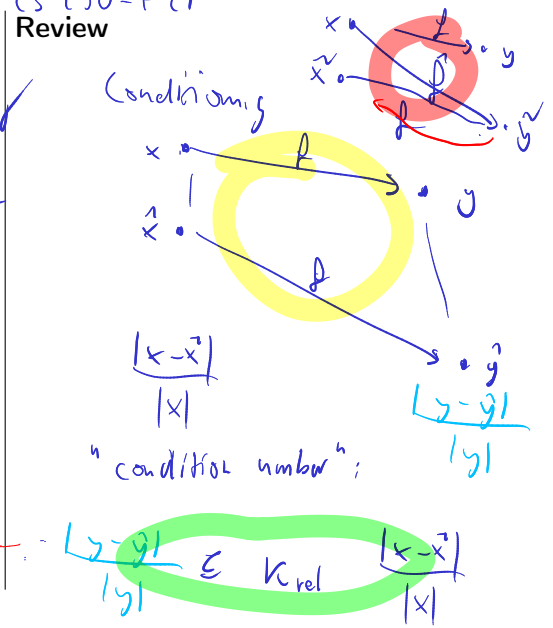
Goals

- Conditioning
- Backward error
- Accuracy / stability
- Floating point

Review



Example: Condition Number of Evaluating a Function

$y = f(x)$. Assume f differentiable.

$$\kappa_{rel} = \max_{x, \hat{x}} \frac{\frac{|f(x) - f(\hat{x})|}{|f(x)|}}{\frac{|x - \hat{x}|}{|x|}}$$

$$f(\hat{x}) = f(\underbrace{x + \Delta x}_{\hat{x}}) = f(x) + f'(x)\Delta x + \dots \quad O(\Delta x^2)$$

$$\kappa_{rel} \geq \frac{|f(\hat{x}) - f(x)|}{|f(x)|} \cdot \frac{|x|}{|x - \hat{x}|} \to \frac{|f(x) + f'(x)\Delta x - f(x)| \cdot |x|}{|f(x)| \quad |\Delta x|} \quad \text{as } \Delta x \to 0$$

$$\sim = \frac{|f'(x)\Delta x| \cdot |x|}{|f(x)| \Delta x} = \frac{|f'(x) \cdot x|}{|f(x)|}$$

Demo: Conditioning of Evaluating tan [cleared]

Stability and Accuracy

Previously: Considered *problems* or *questions*.
Next: Considered *methods*, i.e. computational approaches to find solutions.
When is a method *accurate*?

When is a method *stable*?

Stability and Accuracy

Previously: Considered *problems* or *questions*.
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When is a method *accurate*?

> Closeness of method output to true answer for unperturbed input.

When is a method *stable*?

- "A method is stable if the result it produces is the exact answer for a nearby input."
- The above is commonly called *backward stability*.

  Stricter than: the method's sensitivity to input variation is not much worse than the conditioning.

Relevance of Backward Error

What do we gain from a bound on backward error like

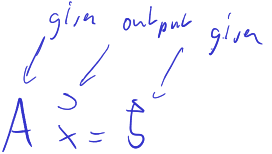
$$\frac{\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|}{\|\boldsymbol{x}\|} \leq \epsilon?$$

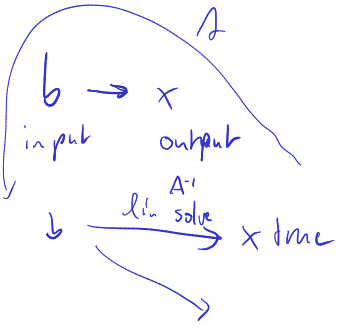
"$f^{-1}(\hat{y})$"

We can use the cond. nr. bound to obtain a statement on (backward) stability.

$$\frac{|y - \hat{y}|}{|y|} \leq \kappa_{cond} \cdot \frac{|x - \hat{x}|}{|x|} \leq \kappa_{rel} \cdot \varepsilon$$

Demo: Backward Stability by Example [cleared]


given
output
given
A
x = b


b → x
input
output
A⁻¹
lin solve
x true

Getting into Trouble with Accuracy and Stability

How can I produce inaccurate results?