Elimination Matrices

- What does this matrix do?

\[
\begin{pmatrix}
1 & 1 \\
-\frac{1}{2} & 1 \\
\end{pmatrix}
\begin{pmatrix}
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
* & * & * & * & * \\
\end{pmatrix}
\]
About Elimination Matrices

- Are elimination matrices invertible?

\[
\begin{bmatrix}
1 & & \\
\frac{1}{2} & & \\
& & \\
& & \\
& & \\
& & \\
\end{bmatrix}^{-1} = \begin{bmatrix}
1 & & \\
\frac{1}{2} & & \\
& & \\
& & \\
& & \\
& & \\
\end{bmatrix}
\]
More on Elimination Matrices

**Demo:** Elimination matrices I

- **Idea:** With enough elimination matrices, we should be able to get a matrix into row echelon form.

\[ M_{q_2} M_{q_1} \cdots M_1 A = U \]

- So what do we get from many combined elimination matrices like that?

**Demo:** Elimination Matrices II
Summary on Elimination Matrices

- El.matrices with off-diagonal entries in a single column just “merge” when multiplied by one another.
- El.matrices with off-diagonal entries in different columns merge when we multiply (left-column) * (right-column) but not the other way around.
- Inverse: Flip sign below diagonal
LU Factorization

- Can build a factorization from elimination matrices. How?
- Does this help solve $Ax = b$?

\[ A = M_e^{-1} M_3 M_2 M_1 A = U \quad \mid \quad M_e^{-1}. \]

\[ A = \underbrace{M_e^{-1}}_{\text{Row echelon form}} U \]

\[ A = L U \quad \rightarrow \quad LUx = b \]

\[ Ly = 0 \quad \underline{\text{solve by forward substitution}} \quad Ux = y \quad \underline{\text{solve by backward substitution}}. \]
Demo: LU factorization

\[ u_n x = b \frac{u_n}{a_{nn}} \]
In-class activity: LU Factorization
LU: Failure Cases?

- Is LU/Gaussian Elimination bulletproof?

- What can be done to get something *like* an LU factorization?
Fixing nonexistence of LU

- How do we capture ‘row switches’ in a factorization?

- What does this process look like then?