

## Recap: Norms

- What's a norm?

$$\|\cdot\|$$

- Define **norm**.

$$\|\gamma \vec{x}\| = |\gamma| \|\vec{x}\|$$

- Examples of norms?

$$\|\cdot\|_p$$

- Does the choice of norm really matter much?

## Demo: Vector norms

## Norms and Errors

- If we're computing a vector result, the error is a vector.  
That's not a very useful answer to 'how big is the error'.  
What can we do?

$$\begin{array}{cc} \vec{x} & \tilde{\vec{x}} \\ \hline \end{array}$$

~~$\|\vec{x}\| = \|\tilde{\vec{x}}\|$~~

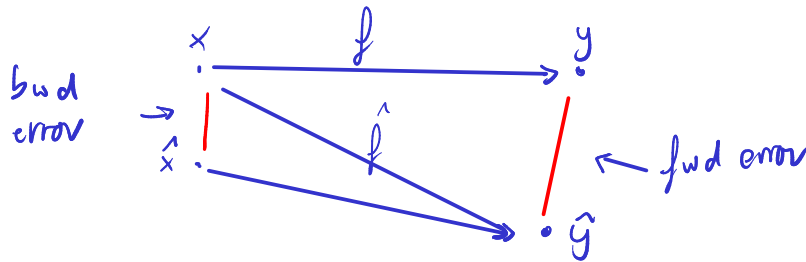
Magnitude of the error:  $\|\vec{x} - \tilde{\vec{x}}\|$

## Forward/Backward Error

- Suppose we're *intending* to compute  $y = f(x)$ , but *actually obtain*  $\hat{y} = \hat{f}(x)$ .

What are the forward error and the backward error?

fwd error:  $\Delta y = y - \hat{y}$



- To find  
bwd error,  
① find  $\hat{x}$   
so that  $f(\hat{x}) = \hat{y}$   
②  $\Delta x = x - \hat{x}$

## Backward Error: Example

- Suppose you wanted  $y = \sqrt{2}$  and got  $\hat{y} = 1.4$ .  
What's the (magnitude of) the forward and backward error?
- What do you observe about the relative errors?

fwd error:

abs.

$$y = \underline{1.4142} \dots$$

$$\hat{y} = \underline{1.4}$$

$$\Delta y = 0.0142$$

rel,

"2 accurate digits"  
↓

$$\frac{\Delta y}{y} = 0.01 \dots$$

backward error:

Find  $\hat{x}$  so that  $\sqrt{\hat{x}} = 1.4 \leadsto \hat{x} = 1.96$

backward error

$$\Delta x = x - \hat{x} = 0.04$$

$$\frac{\Delta x}{x} = \frac{0.04}{2}$$

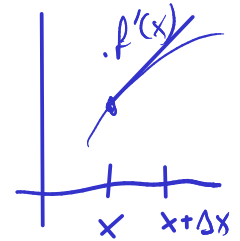
$$= 0.02 \dots$$

**In-class activity:** Forward/Backward Error



## Example: Condition Number of Evaluating a Function

- $y = f(x)$   
Assume  $f$  differentiable.



$$\kappa \approx \frac{|\Delta y| / |y|}{|\Delta x| / |x|} \quad \Delta y = f(x + \Delta x) - f(x) \approx \Delta x \cdot f'(x)$$

$$= \frac{\cancel{|\Delta x|} \cdot |f'(x)| / |f(x)|}{\cancel{|\Delta x|} / |x|} = \frac{|x \cdot f'(x)|}{|f(x)|}$$



**Demo:** Conditioning of evaluating  $\tan()$

**In-class activity:** Conditioning Theory

## Stability and Accuracy

- When is a method **stable**?
- When is a method **accurate**?
- How can I produce inaccurate results?