Elimination Matrices

- What does this matrix do?

$$\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ -\frac{1}{2} & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}$$

$-\frac{1}{2}$

About Elimination Matrices

- Are elimination matrices invertible?

$$\begin{pmatrix} 1 & & & \\ -\frac{1}{2} & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & & & \\ \frac{1}{2} & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$

More on Elimination Matrices

Demo: Elimination matrices I

- **Idea:** With enough elimination matrices, we should be able to get a matrix into row echelon form.

$$M_4 M_3 M_2 M_1 A = U$$

- So what do we get from many combined elimination matrices like that?

Demo: Elimination Matrices II

Summary on Elimination Matrices

- El.matrices with off-diagonal entries in a single column just “merge” when multiplied by one another.
- El.matrices with off-diagonal entries in different columns merge when we multiply (left-column) * (right-column) but not the other way around.
- Inverse: Flip sign below diagonal

LU Factorization

- Can build a factorization from elimination matrices. How?
- Does this help solve $A\mathbf{x} = \mathbf{b}$?

Row echelon form ∇

$$M_\ell \cdots M_4 M_3 M_2 M_1 A = U \quad \Big|\; M_\ell^{-1} \cdot$$

$$A = \underbrace{M_1^{-1} \cdots M_\ell^{-1}}_{\text{lower }\Delta\text{, ones on diagonal}} U$$

$$A = LU \quad \leadsto \quad L\underbrace{Ux}_{y} = b$$

$$\underbrace{Ly = b}_{\text{solve by fw. subst}} \qquad \underbrace{Ux = y}_{\text{solve by bw. subst.}}$$

Demo: LU factorization

$$+ a_{22}x_2 = \frac{b_2 - a_{21} \cdot x_1}{a_{22}}$$

In-class activity: LU Factorization

LU: Failure Cases?

- Is LU/Gaussian Elimination bulletproof?

- What can be done to get something *like* an LU factorization?

Fixing nonexistence of LU

- How do we capture ‘row switches’ in a factorization?

- What does this process look like then?