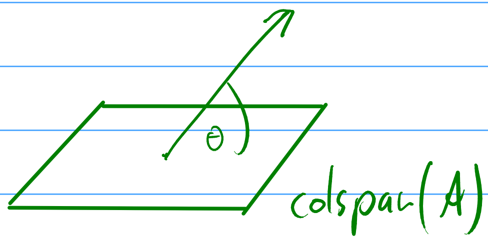
$$\begin{pmatrix} * & * \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} x \cong \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\frac{\|\Delta x\|}{\|x\|} \le \underbrace{\operatorname{cond}(A)}_{3.1} \frac{1}{\cos\theta} \cdot \underbrace{\frac{\|\Delta b\|}{\|b\|}}_{0.1}$$



$Q^T b$

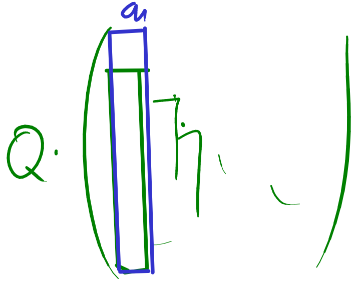
$$\begin{pmatrix} x & x \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} x \approx \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

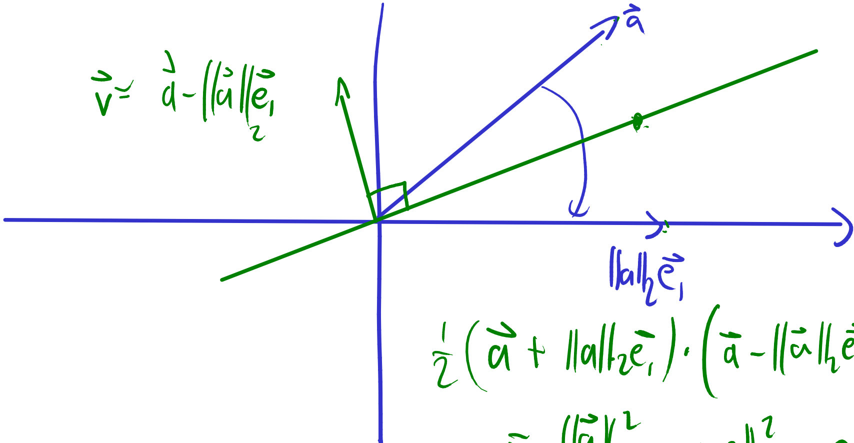
Householder Transformations

- Find an *orthogonal* matrix Q that zeroes out the lower part of a column vector $\boldsymbol{a}$.

Rotate

Reflect





$$H_1 = I - 2\frac{v v^T}{v^T v}$$

$$\vec{v} \cdot \vec{n} = 0$$

$$\frac{1}{2}\left(\vec{a} + \|a\|_2 \vec{e}_1\right) \cdot \left(\vec{a} - \|\vec{a}\|_2 \vec{e}_1\right) = \|\vec{a}\|_2^2 - \|a\|_2^2 = 0$$

$$H_2 = I - \frac{v_2 v_2^T}{v_2^T v_2}$$

- To construct a QR factorization using H_1 $\qquad v = a - \|a\|_2 e_1$

$$H_{n-1} \cdots H_3 H_2 H_1 A = R$$

$$A = \underbrace{H_1 \cdots H_{n-1}}_{Q} R$$

- $\vec{v}_\pm = a \pm \|a\|_2 \vec{e}_1$ both work $\rightarrow$ pick the one that does not encounter cancellation.

- H symm., orthogonal
- Applying Householder

$$\overbrace{\left(I - \frac{vv^T}{v^Tv}\right)}^{O(n^2)} a = \overbrace{\vec{a} - \frac{v(v^Ta)}{v^Tv}}^{O(n)}$$

Demo: 3x3 Householder QR

$$m \times n \quad \begin{pmatrix} \\ A \\ \\ \end{pmatrix}$$

$$\underset{m}{} \begin{pmatrix} \\ Q^T \\ \\ \end{pmatrix}_{n} \begin{array}{c} n \\ \begin{pmatrix} R \\ 0 \end{pmatrix} \end{array} x = \begin{pmatrix} \vec{c} \\ \vec{d} \end{pmatrix}$$

$Q^T b =$

$m \cdot n$

Givens Rotations

- If reflections work, can we make rotations work, too?