

In-class activity: SVDs

$$x = V \Sigma^+ U^T b$$

$$A^+ = V \Sigma^+ U^T \quad \text{"pseudoinverse"}$$

$$x = A^+ b$$

Comparing the Methods

Multiplications to solve least squares with A an $m \times n$ matrix:

- Form: $A^T A$: $n^2 m / 2$ ← $m > n$
Solve with $A^T A$: $n^3 / 6$
- Solve with Householder: $m n^2 - n^3 / 3$
- If $m \approx n$, about the same
- If $m \gg n$: Householder QR requires about twice as much work as normal equations
- SVD: $m n^2 + n^3$ (with a large constant)

Demo: Relative cost of matrix factorizations