Krylov space methods: Intro

- What subspaces can we use to look for eigenvectors?

$$K_n = \begin{bmatrix} x & Ax & A^2x & A^3x & \cdots & A^{n-1}x \end{bmatrix} \rightsquigarrow Q_n R_n = K_n$$

$$AK_n = K_n \begin{pmatrix} 0 & & & \\ 1 & \ddots & & \\ & \ddots & 0 & \\ & & 1 & \end{pmatrix} + \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix} \quad \text{Upper Hessenb.}$$

$$C_n$$

$$K_n^{-1} A K_n = C_n$$

$$Q_n^T A Q_n = R_n C_n R_n^{-1}$$

H

$$AQ_n = Q_n H$$

upper H.

$$Q_n = \begin{pmatrix} | & & | \\ q_1 & \cdots & q_n \\ | & & | \end{pmatrix}$$

$$Aq_k = h_{1k} q_1 + h_{2k} q_2 + \cdots + h_{k+1,k} q_{k+1}$$

$$\vec{q}_j^T A \vec{q}_k = h_{jk}$$

Conditioning in Krylov Space Methods/Arnoldi Iteration

- What is a problem with Krylov space methods? How can we fix it?

Demo: Arnoldi Iteration (Part I)

Krylov: What about eigenvalues?

- How can we use Arnoldi/Lancos to compute eigenvalues?

$$Q = \begin{pmatrix} Q_k & U_k \end{pmatrix}$$

$$H = Q^T A Q = \begin{pmatrix} Q_k^T \\ U_k^T \end{pmatrix} A \begin{pmatrix} Q_k & U_k \end{pmatrix} = \begin{pmatrix} \square & \square \\ \square & \bullet \end{pmatrix}$$

Except for •, eigenvalues of H are a subset of the eigenvalues of A.

With •, they're an approximation to the eigen v. of A.

They are called Ritz values

Demo: Arnoldi Iteration (part 2)
In-class activity: Krylov space methods

Quiz:

$Q_k R_k = X_k$

$X_{k+1} = A Q_k$

$\hookrightarrow$

$Q \underline{R} Q^T \propto A$

D

5 Nonlinear Equations

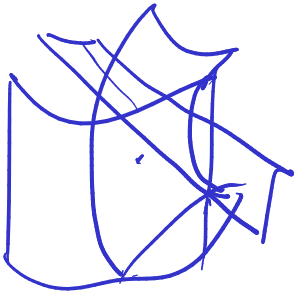
Solving Nonlinear Equations

- What is the goal here?

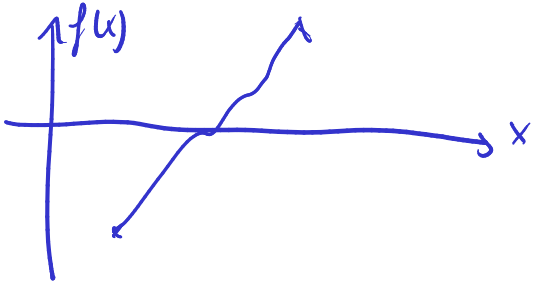
$f(\vec{x}) = \vec{0}$ $\qquad f: \mathbb{R}^n \to \mathbb{R}^n$

#unknowns #equations

$\tilde{f}(\vec{x}) = \vec{y} \rightsquigarrow f(\vec{x}) = \tilde{f}(\vec{x}) - \vec{y}$



Showing Existence

- How can we show existence of a root?
  - 

f continuous

Intermediate value theorem

  - 

Inverse function thm

$f'(x^*) \neq 0$

  - Contraction mapping theorem

$g: \mathbb{R}^n \to \mathbb{R}^n$

$\| g(x) - g(y) \| \leq \gamma \|x - y\| \quad 0 \leq \gamma < 1$

Sensitivity and Multiplicity

- What is the sensitivity/conditioning of root finding?
- What are multiple roots?
- How do multiple roots interact with conditioning?

5.1 Iterative Procedures

- What is linear convergence? quadratic convergence?