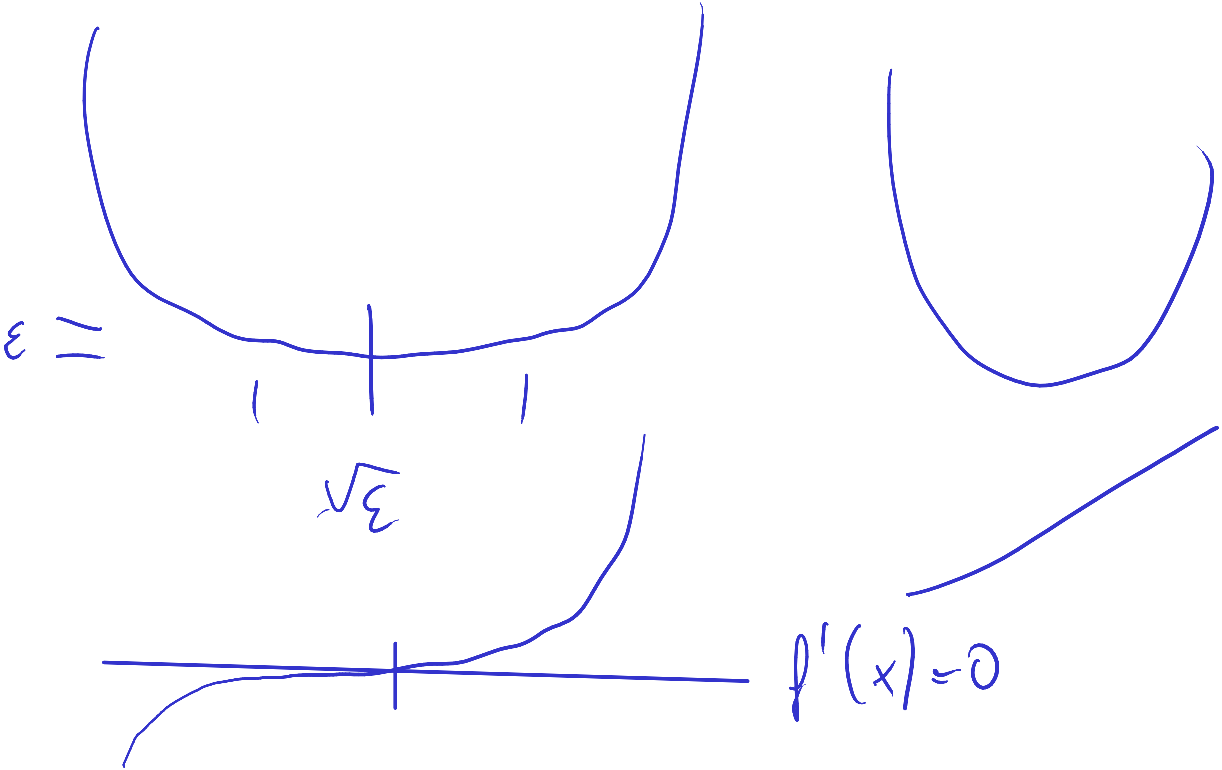
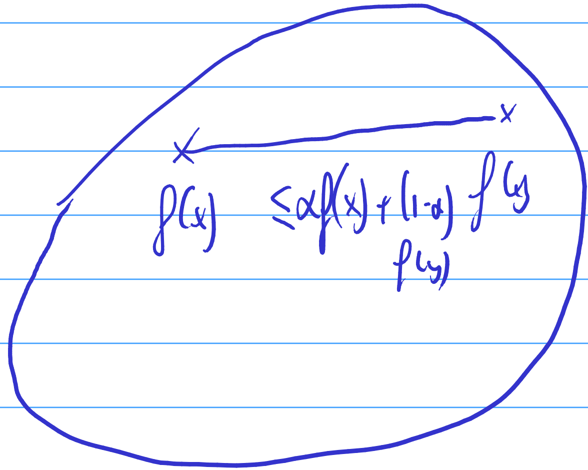
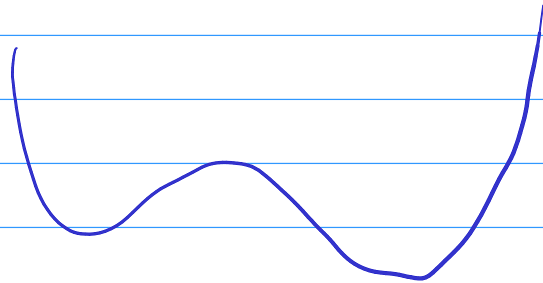
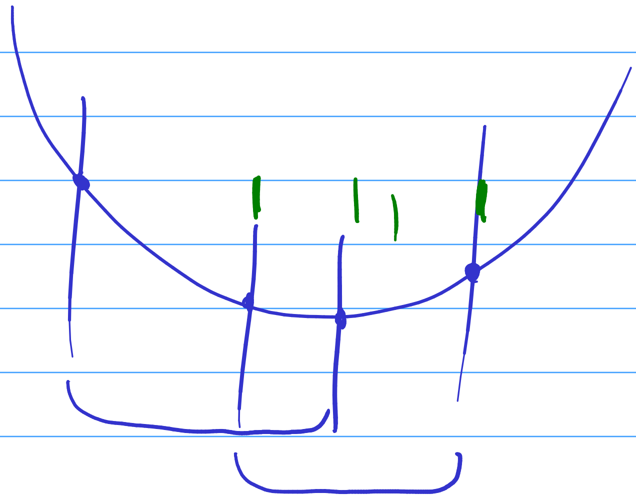
# 6.1 Methods for unconstrained opt. in one dimension




f(x)
≤ αf(x) + (1-α) f(y)
f(y)





## Golden Section Search

- Would like a method like bisection, but for optimization.
  In general: No invariant that can be preserved.
  Need *extra assumption*.

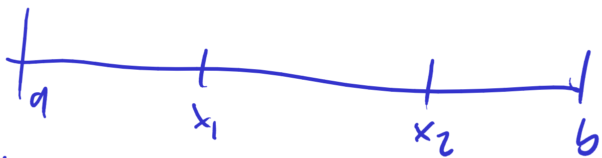
$f$ is called unimodal if for all $x_1 < x_2$

$$x_2 < x^* \Rightarrow f(x_1) > f(x_2)$$

$$x^* < x_1 \Rightarrow f(x_1) < f(x_2)$$

Golden section search:

$a, b$

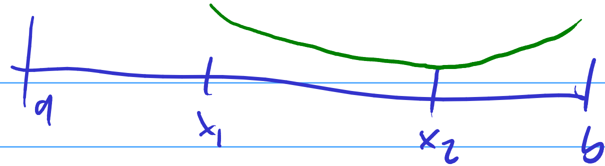



$$x_1 = a + \underbrace{\tau}_{}(b-a) \qquad \tau^2 = 1 - \tau$$

$$x_2 = a + \underbrace{(1-\tau)}_{}(b-a) \qquad \tau = (\sqrt{5}-1)/2$$

If $f(x_1) > f(x_2)$

use $[x_1 \dots b]$




If $f(x_1) \leq f(x_2)$

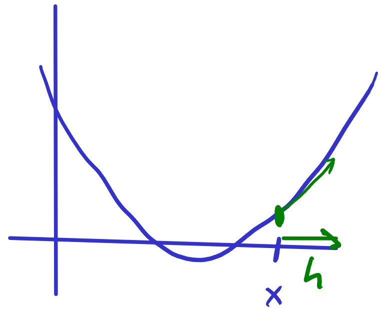
use $[a, x_2]$.

$\rightsquigarrow$ linearly conv.

**Demo:** Golden Section Search Proportions

## Newton's Method

- Reuse the Taylor approximation idea, but for optimization.




$$f(x+h) \approx f(x) + f'(x)h + f''(x)\frac{h^2}{2} = \hat{f}(x+h)$$

$$\frac{\partial}{\partial h}\hat{f}(x+h) = f'(x) + f''(x)\cdot h$$

$$\frac{\partial}{\partial h}\hat{f}(x+h) \overset{!}{=} 0 \overset{!}{=} f'(x) + f''(x)\cdot h$$

$$\rightsquigarrow \quad h = -\frac{f'(x)}{f''(x)}$$

$x_0 = \langle$ some starting guess $\rangle$

$$x_{k+1} = x_k + h = x_k - \frac{f'(x_k)}{f''(x_k)}$$

Newton for opti. $f(x)$

Newton for solving $f'(x) = 0$

$$\frac{f'(x_k)}{f''(x_k)}$$

$$\left[ -\frac{\varphi(x_k)}{\varphi'(x_k)} \right] \qquad \varphi(x) = 0$$

**Demo:** Newton's method in 1D

## 6.2 Methods for unconstrained opt. in $n$ dimensions

## Steepest Descent

- Given a scalar function $f : \mathbb{R}^n \to \mathbb{R}$ at a point $\boldsymbol{x}$, which way is down?