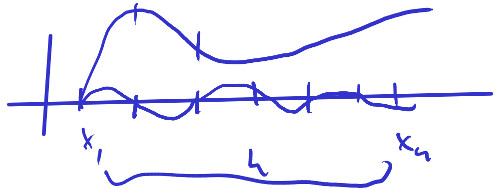
Error Result



$$f(x) - p_{n-1}(x) = \frac{f^{(n)}(\xi)}{n!}(x - x_1)(x - x_2)\cdots(x - x_n)$$

- Why does Chebyshev-like 'bunching' work?

$h = x_n - x_1$

$\rightarrow \max |f^{(n)}(\xi)|$

$$\max |f(x) - p_{n-1}(x)| \leq c \cdot M \cdot h^n$$

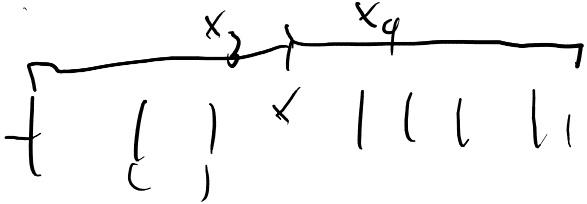
$\text{Error}(h) = O(h^n)$

This is called nth order convergence.

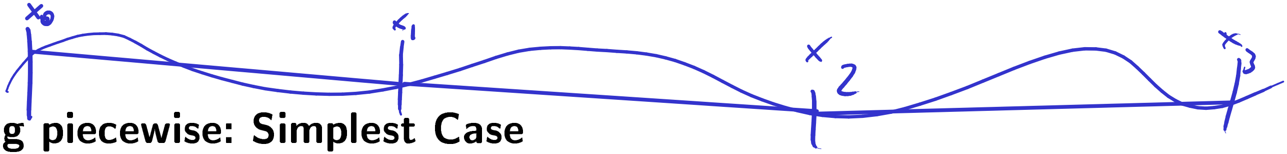
→ ⚠ only holds as $h \to 0$ ("in the asymptotic regime")

→ we really need n derivatives

$(x - x_1) \leq (x_4 - x_3)$



- Boil the error result down to a simpler form.



Going piecewise: Simplest Case

Construct a piecweise linear interpolant at four points.

x_0, y_0		x_1, y_1		x_2, y_2		x_3, y_3
|	$f_1 = a_1 x + b_1$	|	$f_2 = a_2 x + b_2$	|	$f_3 = a_3 x + b_3$	|
|	2 unk.	|	2 unk.	|	2 unk.	|
|	$f_1(x_0) = y_0$	|	$f_2(x_1) = y_1$	|	$f_3(x_2) = y_2$	|
|	$f_1(x_1) = y_1$	|	$f_2(x_2) = y_2$	|	$f_3(x_3) = y_3$	|
|	2 eqn.	|	2 eqn.	|	2 eqn.	|

- ○ Why three intervals?

Piecewise Cubic ('Splines')

Now do the same accounting for piecewise cubic at four points:

x_0, y_0 | $f_1 = a_1x^3 + b_1x^2 + c_1x + d_1$ | x_1, y_1 | $f_2 = a_2x^3 + b_2x^2 + c_2x + d_2$ | x_2, y_2 | $f_3 = a_3x^3 + b_3x^2 + c_3x + d_3$ | x_3, y_3

$\Sigma 12$ — 4 unknowns — 4 unknowns — 4 unknowns

6 eq.

$f_1(x_0) = y_0$

$f_1(x_1) = y_1$

2 — $f_1'(x_1) = f_2'(x_1)$ — $f_2'(x_2) = f_3'(x_2)$

2 — $f_1''(x_1) = f_2''(x_1)$ — $f_2''(x_2) = f_3''(x_2)$

$f''(x_0) = 0$ — $f''(x_3) = 0$

"natural spline"

N # intervals

p # middle points $= N-1$

---

unknowns: $4N$

conditions

	end	middle $(N-2)$	end
Func val	2	2	2
Middle pt. deriv	2	2	
End pt. deriv	1		1

conditions: $2N + 2(N-1) + 2 = 4N$

8 Numerical Integration and Differentiation

8.1 Numerical Integration

Numerical Integration: About the Problem

- What is numerical integration? (Or ‘quadrature’?)

- What about existence and uniqueness?

Conditioning

- Derive the (absolute) condition number for numerical integration.