May 4, 2026

Announcements

- Final
- Exam 4 retake grades
- Review session
- Cheat sheet

Goals

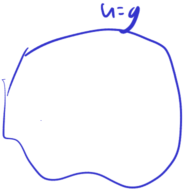
PDEs!

Classification of PDEs

$$a u_{xx} + b u_{xy} + b u_{yx} + c u_{yy} + \text{lower order} = 0$$

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$\Delta u = u_{xx} + u_{yy} = 0 \quad \text{(soap film)}$$



same: elliptic

one differing sign: hyperbolic

One zero: parabolic

time-like: causality

$u_{tt} = u_{xx}$ ("wave")

$u_t = u_{xx}$ ("heat")

Conservation laws

$$u_t + f(u)_x = 0 \quad \leftarrow \text{scalar, 1D}$$

$$\vec{q}_t + \nabla \cdot F(\vec{q}) = 0 \quad \leftarrow \text{system multi-D}$$

- Maxwell's
- Euler (fluid flow)

$$\frac{\partial}{\partial t}\int_a^b u(x,t) + \int_a^b \partial_x f(u)_x = 0$$

$$\cdots + \left[f(u) \right]_a^b = 0$$

- $u_{tt} = u_{xx}$ (wave)
- $u_t + a u_x = 0$ (advection)
- $u_t + \left(\frac{u^2}{2}\right)_x = 0$ (Burgers)

$$\vec{q} = \begin{pmatrix} v \\ w \end{pmatrix} \qquad \partial_t q + \nabla \cdot F(q) = 0 \qquad \vec{q}_t + A \vec{q}_x = 0$$

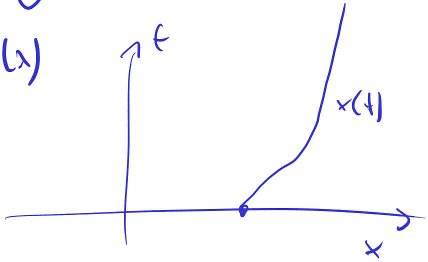
$$\begin{pmatrix} v \\ w \end{pmatrix}_t + \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} v \\ w \end{pmatrix} \right)_x = \vec{0}$$

Fixed after class!

Characteristics

$$u_t + f(u)_x = 0$$

$$u(x,0) = g(x)$$



$$\frac{dx(t)}{dt} = f'(u(x(t),t))$$

$$x(0) = x_0$$

$$\frac{du(x(t),t)}{dt} = u_x x'(t) + u_t$$

$$= u_x f'(u(x,t),t) + u_t = (f(u))_x + u_t = 0$$

Advection

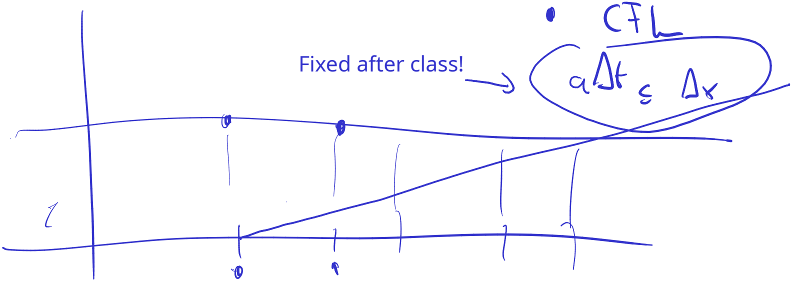
$$u_t + (a u)_x = 0$$

For stability:

- upwinded derivatives
- CFL

Fixed after class!

$$a \Delta t \le \Delta x$$

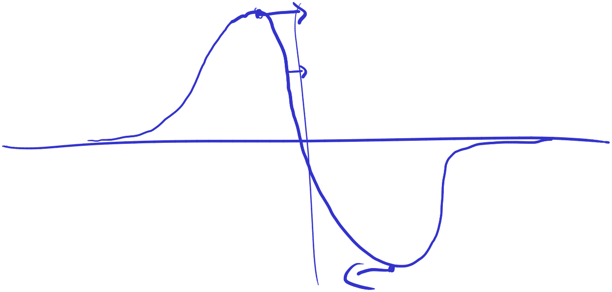


Burgers

$$u_t + \left(\frac{u^2}{2}\right)_x = 0$$

$$f(u) = \frac{u^2}{2}$$

$$f'(u) = u$$



Elliptic

$$\Delta u = 0 \quad \text{"Laplace"}$$

$$\Delta u = f \quad \text{"Poisson"}$$

div grad $u = 0$

$$\nabla \cdot (u\vec{v}) = \nabla u \cdot \vec{v} + u\, \nabla \cdot \vec{v}$$

$$\int \qquad \int \qquad \int$$

trial test

$$a(u,v) = \ell(v)$$

$$\int \Delta u\, v = \int f v$$