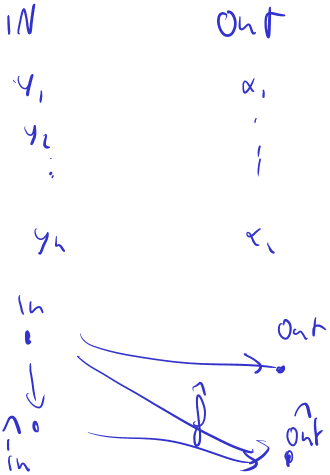
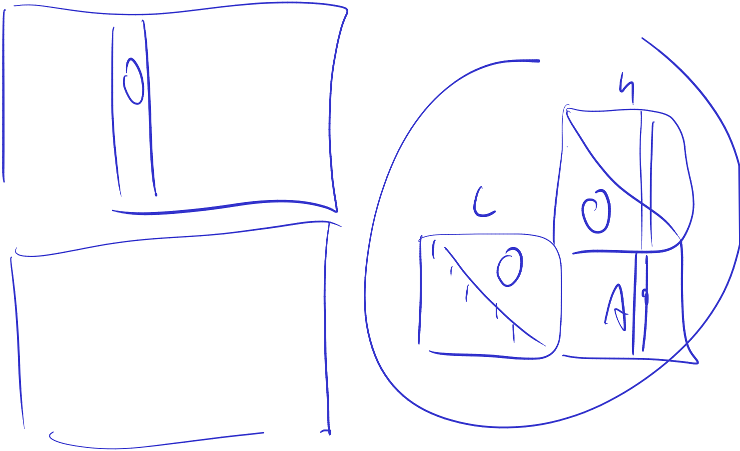
Announcements

- Please fill out FLEX
- HW9 due tonight
- 4CH2 due towards end of finals
- Final starts tomorrow (if you want)
  - 1h50, 16 (8+6+2) MC/entry, 2 code, 17/20 -> 100%
  - Equally weighted
  - Up to and including IVPs, BVPs not on the exam

What does backward stability of the linear solve mean for interpolation?



- What happens with zero columns in LU factorization?
- What happens for non-square?



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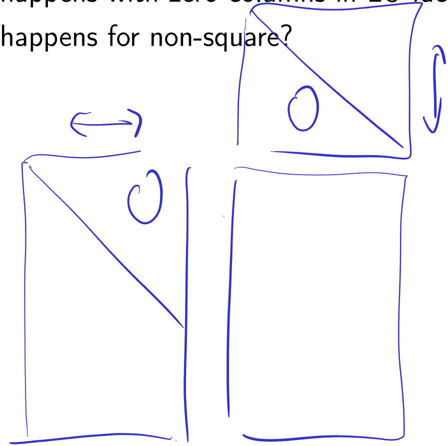


Relate Schur complement and Sherman-Morrison:

$$\begin{bmatrix} A & \boldsymbol{a} \\ -\boldsymbol{a}^\top & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ z \end{bmatrix} = \begin{bmatrix} \boldsymbol{b} \\ 0 \end{bmatrix}$$

u, v

$$(A + uv^T)x = b$$

$$A\vec{x} + \vec{u}z = \vec{b} \iff A\vec{x} + \vec{u}\,v^T\vec{x} = \vec{b}$$

$$-\vec{v}^T\vec{x} + z = 0 \iff z = \vec{v}^T\vec{x}$$

Orthogonality can only be determined up to sqrt(eps_{mach}). Why?

$$\left\| \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix} \right\|_2 \qquad |\varepsilon| < \sqrt{\varepsilon_{mach}}$$

$$= 1 + \varepsilon^2 =_{fl} 1$$

$$\varepsilon^2 < \varepsilon_{mach}$$

Provide a bound on the quality of QR factorization

HH, Givens

$$Q_n \cdots Q_1 A = R$$

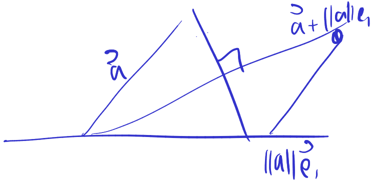
$$\|Q_i\|_2 = 1$$

$$\operatorname{cond}_2(Q_i) = 1$$

$$Q_i \vec{a} = \begin{pmatrix} \pm\|a\|_2 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

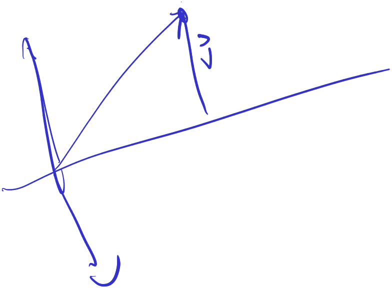
$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix}$$

$$\begin{pmatrix} a & b \end{pmatrix} \cdot \begin{pmatrix} b \\ -a \end{pmatrix} = 0$$



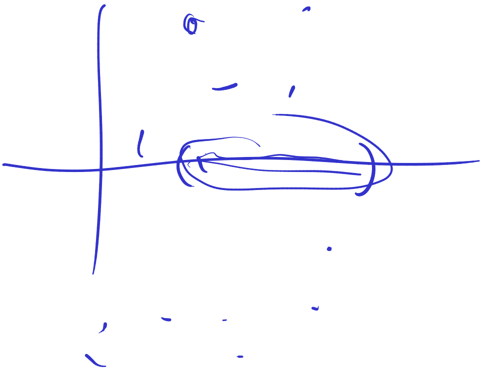
Provide a bound on the quality of QR factorization

$$\text{proj}_{\text{span}(\vec{v})}\vec{x} = \frac{\vec{v}}{\|\vec{v}\|_2}\left(\frac{\vec{v}^T\vec{x}}{\|\vec{v}\|_2}\right)$$



In power iteration on a symmetric matrix, how might you 'kill off' eigenvalues in a whole interval?

- What is a "filter polynomial"?

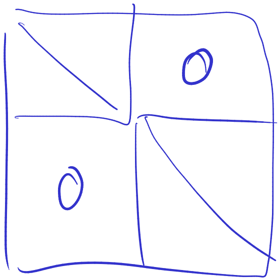


What does a Krylov space buy you? Does it solve any specific problem?

- What's the setting for Krylov space methods?
- How are Kryolv spaces 'just a building block'?
- Explain Krylov usage in eigenvalues and CG.

What does it mean when the Jacobian you encounter in Newton's method is block-diagnoal?

- What is the approximant to the function used by Broyden?

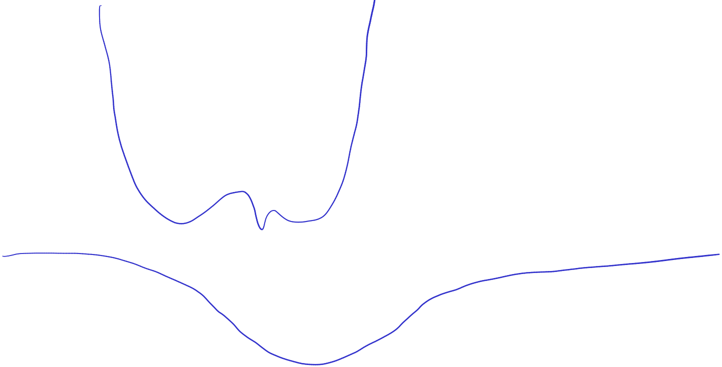


$$f(x_{n+1}) - f(x_n) = \text{"derivative"} \cdot (x_{n+1} - x_n)$$

Look at Broyden update question from exam 3.

- Does convex imply coercive? no
- Does coercive imply convex? no
- Does unimodal imply convex? no

functions



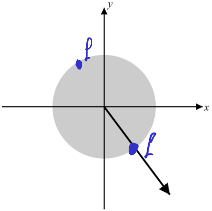
KKT: What do these mean?

$$
\begin{array}{rrcl}
(*) & \nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}^*, \boldsymbol{\lambda}_{\boldsymbol{g}}^*, \boldsymbol{\lambda}_{\boldsymbol{h}}^*) & = & 0 \\
(*) & \boldsymbol{g}(\boldsymbol{x}^*) & = & 0 \\
 & \boldsymbol{h}(\boldsymbol{x}^*) & \geq & 0 \\
 & \boldsymbol{\lambda}_{\boldsymbol{h}}^* & \geq & 0 \\
(*) & \boldsymbol{h}(\boldsymbol{x}^*) \cdot \boldsymbol{\lambda}_{\boldsymbol{h}}^* & = & 0
\end{array}
$$

Do exam 4 inequality-constrained optimization question.

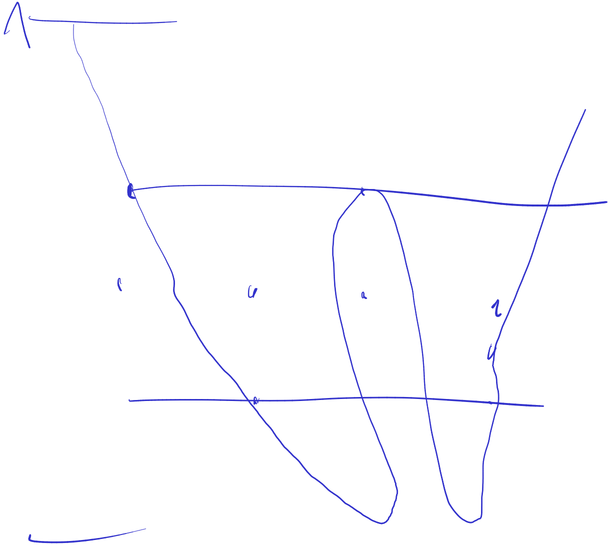
$$L = f - \sum \lambda_{g_i} g_i - \sum \lambda_{h_i} h_i$$

$$\nabla L \stackrel{!}{=} 0$$



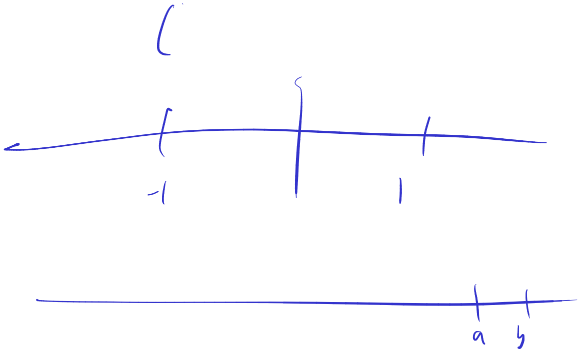
- Interpolate from one grid to another with Vandermonde matrices

- What does a Lebesgue constant of 10,000 mean?



How do I get a differentiation matrix? What do the entries mean?

How do you compute a composite Gaussian integral? (I.e. exam 4 second code question)



Restate the Runge-Kutta setup. Discuss properties.

What's a stability function? How can you use it to check stability (given λ, h)?

Look over inclass 4