April 13, 2026

Announcements

- Exam 4 formula
- Study guide mid-week
- Exam 3 retake grades

Goals

- Gauss $\sim\beta$
- Composite
- Derivatives

Gauss

- For n nodes: n locations, n weights
  - $\Rightarrow$ $2n$ unknowns
  - $\Rightarrow$ $2n$ polys integrated
  - $0, \ldots, 2n-1$ $\leftarrow$ degrees

$(1-x^2)$

Broadcasting

1

(7, 4)

(4, 4)

powers: (4, 1)

nodes: (1, 4)

Composite

$$|\int f - p_{n-1}| = O(h^{n+1})$$

$$\int_1^2 \cdots dx = \underbrace{\sum}_{\frac{1}{h} \text{ terms}} O(h^{n+1}) = O(h^n)$$

Derivatives

on $[0, 2\pi)$

$$\|e^{ix\alpha_n}\|_\infty = 1$$

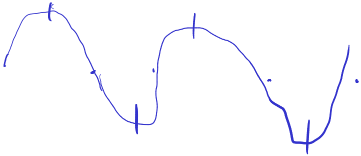
∂_x

$$\|\partial_x e^{ix}\|_\infty = \| i\alpha\, e^{ix\alpha}\|_\infty = |\alpha| \; 1$$

$$\|\partial_x e^{i\alpha x}\|_\infty \leq \underbrace{?}_{\text{none such}} \|e^{i\alpha x}\|_\infty$$

$\Rightarrow$ unbounded

$$p_{n-1}(x) = \sum_{i=0}^{n-1} \alpha \varphi_i(x)$$



$$p'_{n-1}(x) = \sum_{i=0}^{n-1} \alpha \varphi'_i(x)$$

$$\tilde{\alpha} = V^{-1}\tilde{y}$$

$$V' = (\varphi'_j(x_i))_{ij} \qquad D = V'V^{-1}$$

$$(p_{n-1} - f)(x) = \frac{f^{(n)}(\xi(x))}{n!}(x-x_1)\cdots(x-x_n)$$

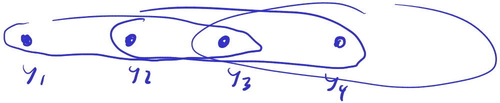
$$\|p'_{n-1} - f'\| \le C \cdot h^n$$

n nodes:

S : h^{n+1}

int : h^{n}

diff : h^{n^4}



$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \times \begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix}$$

$f(\alpha x)$