April 15, 2026

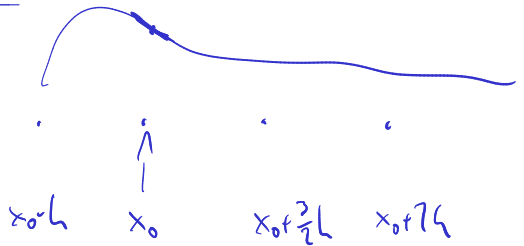
Announcements

- Exam 4
  - cheat sheet
  - study guide
  - next week
- HW7 : due
- Exam 3 page graders out

Goals

- Finite diff via Taylor
- Richardson
- DE
  - ODE
    - IVP setup
    - BVP
  - PDE

Finite difference rules



$V'V^{-1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Richardson

$$F = \tilde{F}(h) + a h^p + O(h^q) \qquad (h \to 0)$$

$$\alpha \tilde{F}(h_1) + \beta \tilde{F}(h_2)$$

h^p power: $\alpha a h_1^p + \beta a h_2^p = 0$

$$\alpha h_1^p + \beta h_2^p = 0 \iff \alpha h_1^p + (1-\alpha) h_2^p = 0$$

$$\alpha + \beta = 1 \iff \beta = 1 - \alpha$$

$$\alpha (h_1^p - h_2^p) = - h_2^p$$

$$\alpha = \frac{h_2^p}{h_2^p - h_1^p}$$

Example:

$$u_1 = \frac{u_0}{2} \qquad p = 1$$

$$\alpha = -\frac{\frac{1}{2}}{1 - \frac{1}{2}} = -1$$

$$k = |1 - (-1)| = 2$$

I VP$_1$

$$y_1' = \alpha y_1 + \beta y_2$$
$$y_2' = \gamma y_1 + \delta y_2$$

↑

$\vec{y}(t) \in \mathbb{R}^n$

explicit

$$y^{(n)}(t) = 12$$

implicit

$$f(y^{(n)}, \dots) = 0$$

$$y^{(n-1)}(t_0) = \dots$$
$$\vdots$$
$$y^{(0)}(t_0) = \dots$$

} Initial conditions

Conversion to first order

$$y'' = -y$$

$$u = y$$
$$v = y'$$

$$\begin{pmatrix} y' \\ y'' \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix}' = \underbrace{\begin{pmatrix} v \\ -u \end{pmatrix}}_{\text{RHS}}$$

Autonomous

$$y''(t) = y(t,t)$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} y \\ t \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \end{pmatrix}' = \begin{pmatrix} u, v \\ 1 \end{pmatrix}$$

$$\tilde{F}(h) = \underbrace{I}_{F} + \left(3h^3 + O(h^4) \right)$$

$$\underbrace{f'(x)}_{G} = \underbrace{\frac{f(x+h) - f(x-h)}{2h}}_{\tilde{F}(h)} + a h^2 + O(h^3)$$