

April 22, 2026

## Announcements

- Exam 4
- 4CH1 deadline  $\rightarrow$  Apr 25
- 4CH2 out today
- HW8 out tomorrow-ish.
- Final
  - equi distrib content
  - longer cheat sheet.

## Goals

- IVP methods
- Stability
- Stiffness
- More methods
- BVPs

# backward Euler

$$y_{k+1} = y_k + h f(y_{k+1})$$

$$y' = \lambda y$$

$$y_{k+1} = y_k + h \lambda y_{k+1}$$

$$y_{k+1} (1 - h\lambda) = y_k$$

$$y_{k+1} = \left( \frac{1}{1 - h\lambda} \right) y_k =$$

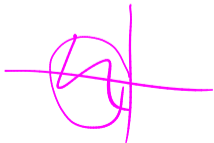
subst:  $h\lambda \rightarrow z$

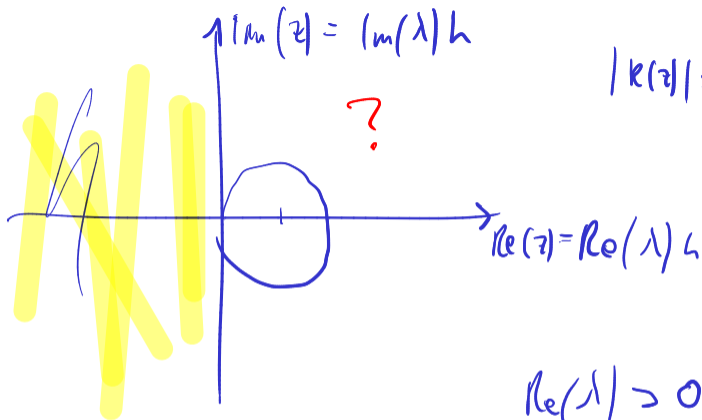
$$R(z) y_k$$

$$\vec{y}' = A \vec{y}$$

$$\vec{y}_{k+1} = \vec{y}_k + h A \vec{y}_{k+1}$$

$$(I - hA) \vec{y}_{k+1} = \vec{y}_k$$





$$|k(z)| \leq 1 \Leftrightarrow \text{stable}$$

$$\text{Re}(\lambda) > 0$$

$$e^{i\alpha} = \cos(\alpha) + i \sin(\alpha)$$

$$e^{-i\alpha} = \cos(\alpha) + i \sin(-\alpha)$$

$$y' = (a+ib)y$$

$$y(t) = e^{(a+ib)t} = e^{at} e^{ibt}$$

(5.1)

# Harmonic oscillator

$$y'' = -y$$

$$\begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \end{bmatrix}' = \begin{bmatrix} v \\ -u \end{bmatrix} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

eigenvalues:  $\pm i$

## More methods

$$y'(t) = f(y)$$

$$y(0) = y_0$$

$$\Rightarrow y(t) = y_0 + \int_0^t f(y(\tau)) d\tau$$

$$y_{n+1}(t) = y_0 + \int_0^t f(y_n(\tau)) d\tau$$

Predictor - Corrector

FE

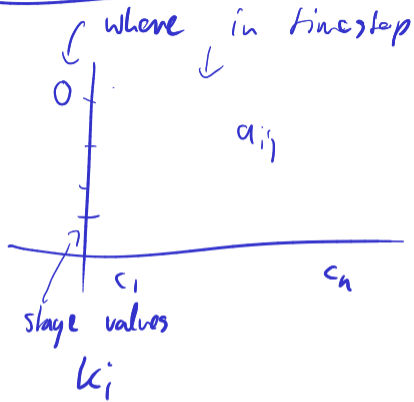
Trapezoidal

}

$\Rightarrow$

Heun

# Butcher tableau



$$k_i = f(y_n + h \sum_{j=1}^n a_{ij} k_j)$$

$$y_{n+1} = y_n + h \sum_{i=1}^n c_i k_i$$