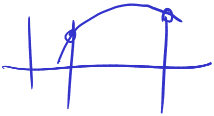
March 25, 2026

Announcements

- Page grader
  - Exam 2
  - Retake

Golden section search



Goals

n-dim optimization

Steepest descent

Heavy ball etc.

CG

(Nelder-Mead)

Newton

BFGS.

Steepest descent

$$f : \mathbb{R}^n \to \mathbb{R}$$

$$\vec{p}_k = -\nabla f(\vec{x}_k) \quad \text{descent direction}$$

$$\vec{x}_{k+1} = x_k + \alpha_k \vec{p}_k$$

$$f(\vec{x}) = \frac{1}{2} \vec{x}^T A \vec{x} + \vec{x}^T \vec{b}$$

$$\nabla f = Ax + b$$

CG:

$$x_{k+1} = x_k - \alpha \nabla f(x_k) + \beta \underbrace{(x_k - x_{k-1})}_{\bar{s}_k}$$

Problem $\quad f(\vec{x}) = \frac{1}{2} \vec{x}^T A \vec{x} - \vec{x}^T \vec{b}$

$$\nabla f(\vec{x}) = A\vec{x} - \vec{b} = \vec{r}$$

$$\nabla f \stackrel{!}{=} 0$$

$$\left.\begin{aligned} \bar{r}_{k+1}^T \vec{s}_k &= 0 \\ \vec{r}_{k+1} \nabla f(\vec{x}_k) &= 0 \end{aligned}\right\} \quad \dots \quad \bar{s}_i^T A \vec{s}_j = 0 \quad (i \neq j)$$

Wolfe conditions

- "f has decreased enough"
- "Slope in descent direction must be 'less steep' than at $\vec{x}_k$"

$$\nabla f(x_k + \alpha_k \vec{p}_k)^T \vec{p}_k \geq c \nabla f(\vec{x}_k)^T \vec{p}_k$$

Quasi - Newton

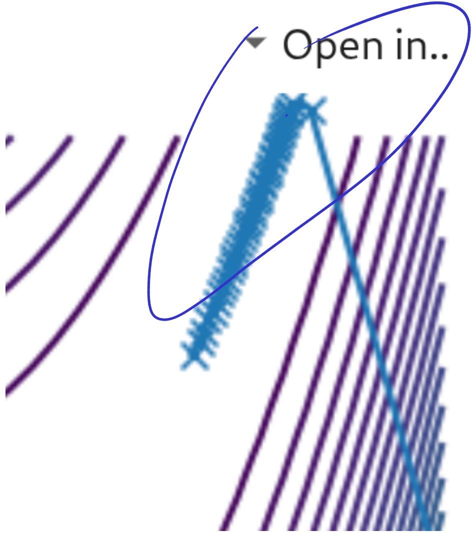
"Hessian"

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - \alpha_k B_k^{-1} \nabla f(\boldsymbol{x}_k)$$

- α_k: a line search/damping parameter.
- $\boldsymbol{s}_k = \boldsymbol{x}_{k+1} - \boldsymbol{x}_k$
- $\boldsymbol{y}_k = \nabla f(\boldsymbol{x}_{k+1}) - \nabla f(\boldsymbol{x}_k)$
- Secant condition: $B_{k+1}\boldsymbol{s}_k = \boldsymbol{y}_k$
- Ansatz for Hessian update: $B_{k+1} = B_k + a\boldsymbol{u}\boldsymbol{u}^T + b\boldsymbol{v}\boldsymbol{v}^T$

$y_k^T s_k > 0$

$$B_{k+1} = B_k + \frac{\boldsymbol{y}_k \boldsymbol{y}_k^T}{\boldsymbol{y}_k^T \boldsymbol{s}_k} - \frac{B_k \boldsymbol{s}_k \boldsymbol{s}_k^T B_k}{\boldsymbol{s}_k^T B_k \boldsymbol{s}_k}$$



$x^T A x^T \sim \alpha x^2 + \beta y^2 + \ldots$