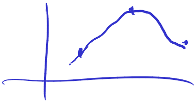
April 1, 2026

Announcements

- Exam 3 → grades Fri ?
- Exam 3 second
- HW6

Goals

- Interpolation



$$p_{n-1}(x) = \sum_{i=0}^{n-1} \alpha_i \varphi_i(x)$$

$$y_j = p_{n-1}(x_j) = \sum_{i=0}^{n-1} \alpha_i \varphi_i(x_j)$$

V_{ji}

equi + x^j ; Vandermonde

"generalized"

What do the numbers mean?

- model coefficients $\vec{\alpha}$
- function values at fixed nodes: $\vec{y}$

$$V \vec{\alpha} = \vec{y}$$

Hard in coefficients:

$$(p_{n-1})^2 = \left(\sum \ldots \right)^2$$

Hard in pointwise

$$p_{n-1}'(x) = \sum_{i=0}^{n-1} \alpha_i \varphi_i'(x)$$

$$V_1 \left(V_2^{-1} \vec{y} \right)$$

square

$$V_2^{-1} \vec{y} \rightarrow \text{coefficints}$$

lots of nodes?

$$D = V' V^{-1} \vec{y}$$

Menu

Basis

x^j

$\sin(nx)$

$e^{inx} = (e^{ix})^n$

Nodes

equispaced

ℓ^p

$$\|\vec{x}\|_p = \sqrt[p]{\sum_i |x_i|^p}$$

$p=\infty$: max

L^p

$$\|f\|_p = \sqrt[p]{\int |f(x)|^p}$$

$p=\infty$: max

$$\| \quad \|_\infty$$

Sensitivity

- $\kappa(V)$
- $\| p_{n-1}^{\vec{y}} \|_\infty \leq \underbrace{\Lambda}_{\text{Lebesgue constant}} \| \vec{y} \|_\infty$

$\tilde{\vec{y}} = \vec{y} + \vec{\delta}$

$$\| p_{n-1}^{\vec{\delta}} \|_\infty \leq \Lambda \| \delta \|_\infty$$

Best-approx. $\swarrow$ Interp. operator

$$Pf = p_{n-1}$$

Let q be any polynomial of degree $n-1$.

$$Pq = q$$

Both P and Λ depend on nodes and the basis.

$$\|f - Pf\|_\infty = \|f - q + q - Pf\|_\infty$$

$$\leq \|f - q\|_\infty + \|q - Pf\|_\infty$$

$$= \|f - q\|_\infty + \|P(q - f)\|_\infty$$

$$\leq \|f - q\|_\infty + \Lambda \|f - q\|_\infty = (1+\Lambda)\|f - q\|_\infty.$$

Lagrange $\quad V = I \leftarrow$ Goal

$$\varphi_j(x_i) = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

$$\varphi_1(x_1) \stackrel{!}{=} 1$$

$$\varphi_1(x) = (x - x_2)(x - x_3) / (x_1 - x_2)(x_1 - x_3)$$

$$\varphi_2(x) = (x - x_1) \quad (x - x_3)$$

$$\varphi_3(x) = (x - x_1)(x - x_2)$$

$$p_{n-1}(x) = \underbrace{y_1}\varphi_1(x) + \underbrace{y_2}\varphi_2(x) + \underbrace{y_3}\varphi_3(x)$$