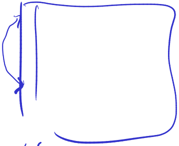
February 9, 2026

Announcements

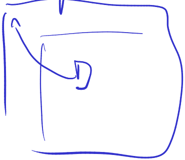
- Exam 1
  - Study guide
  - Book review Q's
- HW2, exam 1 second coming
- SVD / 2-norm: see forum

Goals

partial piv:



complete piv:



$$(A + \vec{u}\vec{v}^T)^{-1}$$

$\uparrow$ rank 1

partial: $PA = LU$

complete: $PAQ = LU$

$\uparrow \nearrow$ permutation matrices

Schur complement

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

Operating on blocks can be way less expensive than single rows/columns

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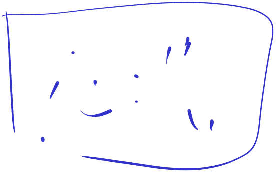
Cost: How do you save?

- Sparse



- Low rank

$$\begin{array}{c} \\ m \end{array}\overset{n}{\boxed{A}} = \begin{array}{c} \\ m \end{array}\overset{k}{\boxed{U}}\;\overset{n}{\boxed{V^T}}\,\}k$$

$$A\tilde{x} = \cancel{(UV^T)x} \leftarrow m \cdot n \cdot k$$

$$= U\underbrace{(V^Tx)}_{n \cdot k \text{ cost}}$$

$$\qquad\qquad = y$$

$$= Uy \quad \} mk$$

$$mk + nk$$

Cost: $C \cdot n^3 + O(n^2)$

Cholesky

$A = A^T$

$= LU$

1s on the diagonal

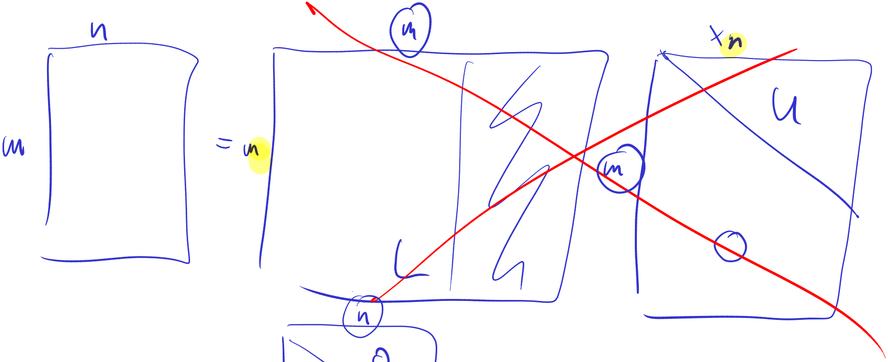
$A^T = LL^T$

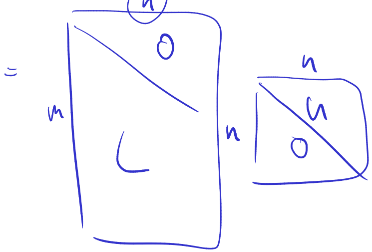
SPSD

SPD: $\vec{x}^T A \vec{x} > 0$ $(\vec{x} \neq \vec{0})$

SPSD: $\vec{x}^T A \vec{x} \geq 0$

Cost:



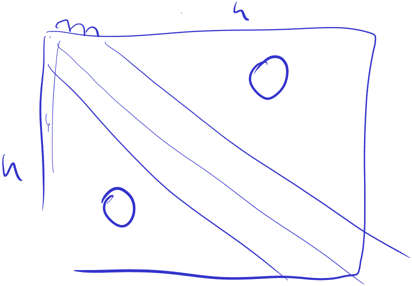


"economy" factorization

$A\vec{x}=\vec{b}$ → LU / Fw / Bw = $O(n^3)$

$AX=B$ → LU / n x Fw / n x Bw = $O(n^3)$

$AX=I$ → find an inverse



→ $O(n\beta^2)$

Basic Linear Algebra Subroutines

$$A_{ij} = \sum_{k} B_{ik} C_{kj}$$

$\begin{pmatrix}\varepsilon & 1\\ 1 & 1\end{pmatrix}$

unpiv.

- Without pivoting: $L = \begin{bmatrix} 1 & 0 \\ 1/\epsilon & 1 \end{bmatrix}$, $U = \begin{bmatrix} \epsilon & 1 \\ 0 & \cancel{1} - 1/\epsilon \end{bmatrix}$
- Rounding: $\mathrm{fl}(U)) = \begin{bmatrix} \epsilon & 1 \\ 0 & -1/\epsilon \end{bmatrix}$
- This leads to $L\,\mathrm{fl}(U)) = \begin{bmatrix} \epsilon & 1 \\ 1 & 0 \end{bmatrix}$, a backward error of $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

piv.

- We now compute $L = \begin{bmatrix} 1 & 0 \\ \epsilon & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 1 \\ 0 & 1 - \epsilon \end{bmatrix}$, so

  $\mathrm{fl}(U) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
- This leads to $L\,\mathrm{fl}(U) = \begin{bmatrix} 1 & 1 \\ \epsilon & 1 + \epsilon \end{bmatrix}$, a backward error of

  $\begin{bmatrix} 0 & 0 \\ 0 & \epsilon \end{bmatrix}$.

$$\hat{A} = A + \boldsymbol{u}\boldsymbol{v}^T$$

n^2

n

n

n^2

$$(A + \boldsymbol{u}\boldsymbol{v}^T)^{-1} = A^{-1} - \frac{A^{-1}\boldsymbol{u}\boldsymbol{v}^T A^{-1}}{1 + \boldsymbol{v}^T A^{-1}\boldsymbol{u}} \cdot \vec{x}$$

n

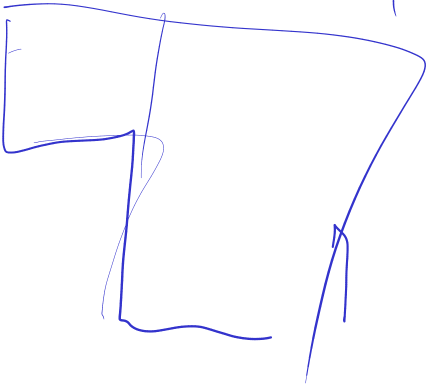
$\vec{x} = A^{-1}\vec{x} -$

n^2

$O(n^2)$

$(A^{-1}\vec{x})$

$$[Ax]_i = \sum A_{ij} x_j$$



$$A^{-1}x = (LU)^{-1}x = U^{-1}(L^{-1}x)$$