February 16, 2026

Announcements

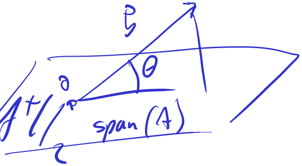
- Exam 1 graded, 78
  - Timer: use PT
  - formula sheet (ex.2 ff)
  - make use of helpdesk.
- 1e-5 `10^(-5)` ← bitwise xor

  `10**(-5)`

Goals

- CSQ sensitivity
- QR for CSQ
- Computing QR via Gram-Schmidt

$\kappa(A) = \|A\|_2 \, \|A^+\|_2$



out

in

$$\begin{aligned}
\frac{\|\Delta \boldsymbol{x}\|_2}{\|\boldsymbol{x}\|_2} &\le \|A^+\|_2 \frac{\|\Delta \boldsymbol{b}\|_2}{\|\boldsymbol{x}\|_2} \\
&= \frac{\kappa(A)}{\|A\|_2 \, \|A^+\|_2} \|A^+\|_2 \frac{\|\boldsymbol{b}\|_2}{\|\boldsymbol{b}\|_2} \frac{\|\Delta \boldsymbol{b}\|_2}{\|\boldsymbol{x}\|_2} \\
&= \kappa(A) \frac{\|\boldsymbol{b}\|_2}{\|A\|_2 \, \|\boldsymbol{x}\|_2} \frac{\|\Delta \boldsymbol{b}\|_2}{\|\boldsymbol{b}\|_2} \le \kappa(A) \underbrace{\frac{\|\boldsymbol{b}\|_2}{\|A\boldsymbol{x}\|_2}}_{1/\cos\theta} \frac{\|\Delta \boldsymbol{b}\|_2}{\|\boldsymbol{b}\|_2}.
\end{aligned}$$

$$\frac{\|\Delta x\|}{\|x\|} \Big/ \frac{\|\Delta b\|}{\|b\|} \le \mathrm{cond}(LSQ)$$

$\|b\|_2$

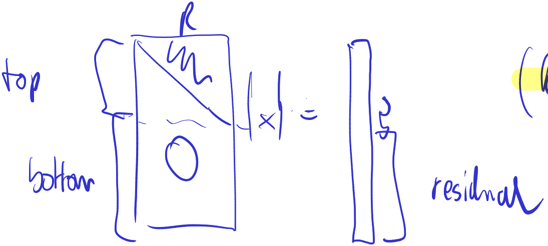
$\overset{m\times n}{A} = \overset{m\times m}{Q}\underset{m\times n}{R}$ $\quad m>n \quad$ $Q^TQ = QQ^T = I$

Full ; i.e. not economy QR

$$\|Ax-b\|_2 = \|QRx-b\|_2$$

$$= \|Q^T(QR\tilde{x}-\tilde{b})\|_2$$

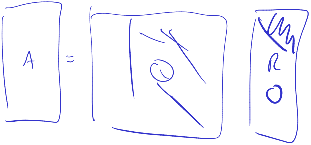
$$= \|R\tilde{x}-Q^Tb\|_2$$



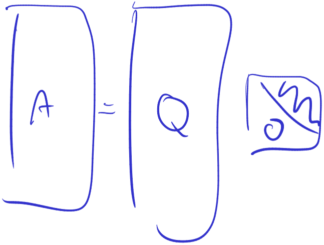
$(R_{top})x = (Q^T b)_{Top}$

economy QR suffices

non-economy



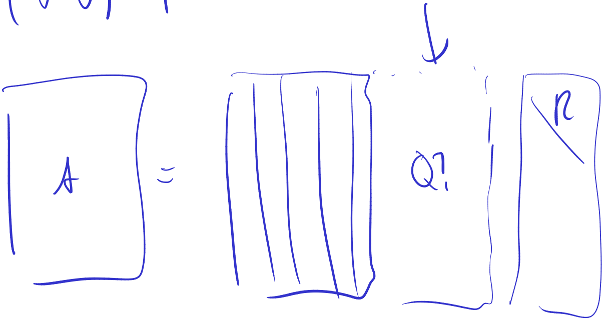
economy:



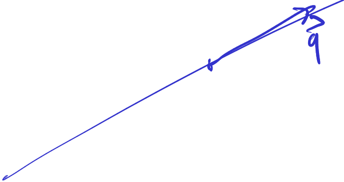
$\vec{v} = \begin{pmatrix} 1 \\ 10^{-8} \end{pmatrix}$ $\vec{w} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\vec{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\vec{v}^T \vec{v} = 1 + 10^{-16}$$

$$fl(v^T v) = 1$$



2D



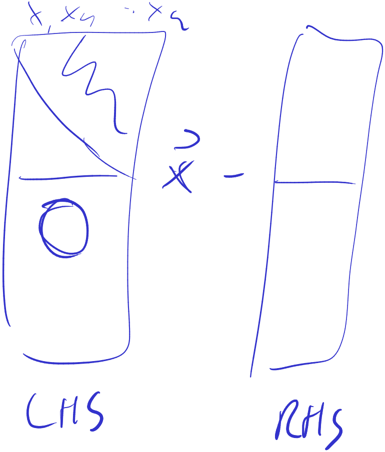
$A^{\sim} = A + \Delta A$

Q, A

$$\begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix}$$

$\varepsilon : |\cdot| \leq \sqrt{\varepsilon_{ma}}$

$$\begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix}$$


x₁ x₄ ·· xₙ
0
x̂ –
LHS
RHS