January 26, 2026

Announcements

In-class / prelecture / exams → "testable nugget"

hw → practical applicability

" 10^{-16} " relative error from dp floating point.

Goals

norm equiv.

fw / bw error

conditioning

acc / stab

bw error

bw stability

"Accurate significant digits"

10,000 ← true → 0.0010000

10,001 ← approx → 0.0010001

4 digits

4 digits

$$\# \text{ accurate sig digits} = -\log 10\left(\frac{|\hat{x}-x|}{|x|}\right)$$

1 ← true

1.3 ← approx

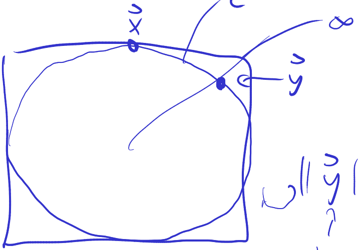
1 ← true

1.7 ← approx

2 true

1.9999

Norm equivalence



$$\underbrace{\|\vec{y}\|_\infty}_{\frac{1}{\sqrt{2}}} \le \|\vec{y}\|_2 \le \underbrace{\sqrt{2}}_{\frac{1}{\sqrt{2}}}\|\vec{y}\|_\infty$$

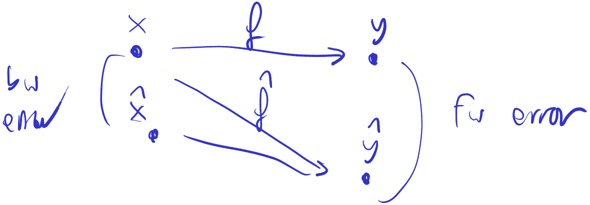
$$\|\vec{x}\|_\infty \le \|\vec{x}\|_2 \le \|\vec{x}\|_\infty$$

$$\|\vec{x}\|_2 = 1 = \|\vec{x}\|_\infty$$

$$\|\vec{y}\|_2 = 1 \qquad \|\vec{y}\|_\infty = \frac{1}{\sqrt{2}}$$

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

Fw / bw error



$$\hat{y} = \frac{1}{2+\hat{x}}$$

$$\Leftrightarrow \frac{1}{\hat{y}} = 2 + \hat{x} \qquad \frac{1}{\hat{y}} - 2 = \hat{x}$$

$$\frac{|f(x) - f(\hat{x})|}{|f(x)|} \leq \kappa_{\text{rel}} \cdot \frac{|x - \hat{x}|}{|x|}$$

$$\kappa_{\text{rel}} = \max_{x,\hat{x}} \frac{|f(x) - f(\hat{x})| \,/\, |f(x)|}{|x - \hat{x}| \,/\, |x|}.$$

$x \in X$

$\hat{x} \in \hat{X}$

$$\frac{\Delta y / y}{\Delta x / x} \to \frac{f'(x)\,\cancel{\Delta x} / f(x)}{\cancel{\Delta x} / x} = \frac{f'(x)\,x}{f(x)}$$

$$\Delta y \to f'(x)\,\Delta x \qquad (\Delta x \to 0)$$

(not technically attainable using sets, a lower bound!)