February 25, 2026

Announcements

- Exam 2
  - contact cut-off ≤ 2/23 (Monday)
  - study guide
  - formula sheet
- HW 3

Goals

- sensitivity of eigval problem
- power iteration
- Schur form

$X^{-1}AX = D$

$X = [\,|\,|\,|\,|\,]$ eigenvectors

$$X^{-1}(A+E)X = D + F$$ similar!

- $A + E$ and $D + F$ have same eigenvalues
- $D + F$ is not necessarily diagonal

Suppose $\boldsymbol{v}$ is an eigenvector of $D + F$.

WLOG $\mu \notin \operatorname{diag}(D)$

$$
\begin{aligned}
& (D+F)\boldsymbol{v} = \mu \boldsymbol{v} \\
\Leftrightarrow\quad & F\boldsymbol{v} = (\mu I - D)\boldsymbol{v} \\
\Leftrightarrow\quad & (\mu I - D)^{-1} F\boldsymbol{v} = \boldsymbol{v} \qquad \text{(when is that invertible?)} \\
\Rightarrow\quad & \|\boldsymbol{v}\| \leq \|(\mu I - D)^{-1}\| \, \|F\| \, \|\boldsymbol{v}\| \qquad \mid \ : \|v\| \\
\Rightarrow\quad & \|(\mu I - D)^{-1}\|^{-1} \leq \|F\|
\end{aligned}
$$

$F = X^{-1}EX$

$\|F\| \leq \operatorname{cond}(X)\|E\|$

$$B = XAX^{-1} \Leftrightarrow X^{-1}AX = B$$

$$A\vec{x} = \lambda\vec{x} \qquad (\vec{x} \neq 0)$$

$$\vec{y} := X^{-1}\vec{x}$$

$$B\vec{y} = X^{-1}A\cancel{XX^{-1}}\vec{x} = X^{-1}A\vec{x} = X^{-1}\lambda\vec{x} = \lambda\vec{y}$$

$$\vec{x} = \alpha x_1 + \beta x_2 + \dots$$

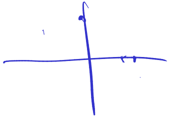
$$A\vec{x}_1 = \lambda_1\vec{x}_1 \qquad (\vec{x}_1 \neq 0)$$

$$A\vec{x} = \alpha\lambda_1\vec{x}_1 + \beta\lambda_2\vec{x}_2 + \dots$$

$$A\vec{x}_2 = \lambda_2\vec{x}_2 \qquad (\vec{x}_2 \neq 0)$$

$$|\lambda_1| \geq |\lambda_2| \geq 0$$

$$\frac{A^n x}{\lambda_1^n}\vec{x} = \alpha\underbrace{\cancel{\left(\frac{\lambda_1}{\lambda_1}\right)^n}}_{1}\vec{x}_1 + \beta\left(\frac{\lambda_2}{\lambda_1}\right)^n\vec{x}_2 + \dots$$



Shift-invert

$$(A - \sigma I)^{-1}$$

↪ converges to eigenvec for closest-to-σ eigenvalue

Rayleigh quotient

$$\frac{\vec{x}^T A \vec{x}}{\vec{x}^T \vec{x}}$$

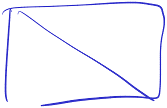
Schur Form

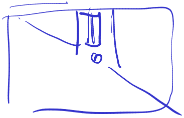
$A = LU$

$\cancel{A = XDX^{-1}}$

$A = Q$  Q^T "Schur form"

U

$U =$ 

$U - \lambda I =$ 

$$\left\| \begin{pmatrix} 1 \\ 1e-8 \end{pmatrix} \right\|_2$$

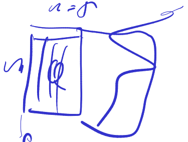
$A = QR$

$Q^T A = R$

$Q^T Q = I$ ✓

$\cancel{QQ^T = I}$ (I: $m \times m$)

$1 = \|I\| = \|AA^{-1}\|$
$\leq \|A\| \, \|A^{-1}\|$
$= \mathrm{cond}(A)$



$q_1 \cdots q_n \in \mathbb{R}^m$

$q_i^T q_j = \begin{cases} 1 & i=j \\ 0 \end{cases}$