April 6, 2026

Announcements

- Exam 3 second
  - Exam 3
  - 4CH1 out tomorrow

Goals

Orth poly

Interp. error

Orth. poly

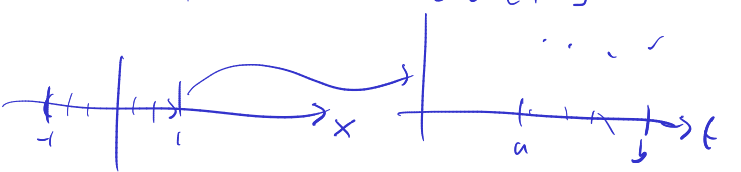
$\Omega = [-1, 1]$

$$(\vec{x}, \vec{y}) = \sum_{i=1}^{n} x_i y_i = \vec{x}^T \vec{y}$$

$$(f, g) = \int_{\Omega} f(x)\, g(x)\, w(x)\, dx$$

$1, x, x^2, x^3, \ldots$

$\tilde{\Omega} = [a, b]$



$$t(x) = \frac{a+b}{2} + \frac{(b-a)}{2} x$$

Chebyshev interp.

$$T_n(x) = \cos\left(n \cos^{-1}(x)\right)$$

$$x_i = \cos\left(\frac{i}{n}\pi\right) \qquad (i = 0 \dots k)$$

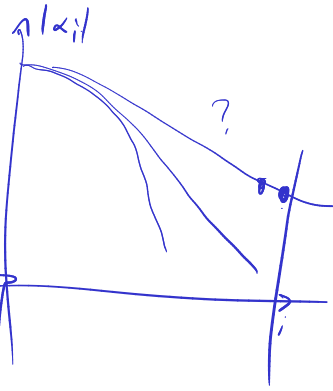
$$V_{ij} = T_j(x_i) = \cos\left(j \cdot \frac{i\pi}{n}\right)$$

$$\downarrow$$

DFT $\leftarrow O(n \log n)$ cost

Orthh poly coefficients

$$f(x) \approx \sum_{i=0}^{n+1} \alpha_i \, P_i(x)$$



$$\sqrt{\left(\sum_{i=0}^{n+1} \alpha_i \, P_i(x), \sum_{i=0}^{n+1} \alpha_i \, P_i(x) \right)}$$

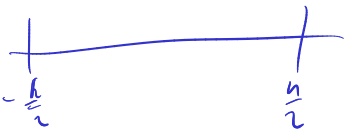
$$= \sqrt{\sum |\alpha_i|^2 (P_i, P_i)} = \sqrt{\sum |\alpha_i| \, \|P_i\|^2}$$

Interpolation error

$$f(x) - p_{n-1}(x) = \frac{f^{(n)}(\xi(x))}{n!} (x - x_1) \cdots (x - x_n)$$

Taylor:

$$\frac{f^{(n)}(a)}{n!} (x-a)^n$$



$$|x - x_i| \leq h$$

$$\| f - p_{n-1} \|_\infty \leq \frac{\| f^{(n)} \|}{n!} \cdot h^n$$