April 8, 2026

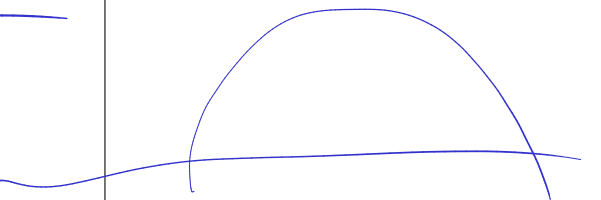
Announcements

- 4CH11
- HW6

(OP recap)

Goals

- Splines / piecewise
- $\int$ "quadrature"



Piecewise

↳ Splines

Cubic

$$\underbrace{p_i}_{ax^3+bx^2+cx+d} \quad \underbrace{p_{i+1}}$$

$$N_{intervals} = N$$

$$N_{interior\ nodes} = M = N-1$$

#unk: $4N$

#cond: Hit y at end point. $2N$

$$\left.\begin{aligned} p_i'(x_{i+1}) &= p_{i+1}'(x_{i+1}) \\ p_i''(x_{i+1}) &= p_{i+1}''(x_{i+1}) \end{aligned}\right\} 2M$$

$4N-2$

"Natural"

$$p_0''(x_0) = 0$$

$$p_{N-1}''(x_N) = 0$$

Quadrature

$$\int_a^b f(x)\,dx = \sum_{i=1}^{n} \omega_i f(x_i)$$

ω_i: quad. rule; x_i

Finding rules:

- $\omega_i = \int_a^b l_i(x)\,dx$

$$p_{n-1}(x) = \sum_{i=1}^{n} f(x_i)\, l_i(x)$$

$$\int - n -$$

$\cdot f(x) \approx p_{n+1}(x) = \sum_{i=0}^{n+1} \alpha_i \varphi_i(x)$ $\cdot(x)$

$V\vec{\alpha} = \vec{y} \Leftrightarrow \vec{\alpha} = V^{-1}\vec{y}$

$\vec{n}_i = \int_a^b \varphi_i(x)\,dx$

$\int_a^b p_{n+1}(x)\,dx = \sum_{i=0}^{n-1} \alpha_i \int_a^b \varphi_i(x)\,dx = \vec{n}^T \vec{\alpha} = \underbrace{\left(\vec{n}^T V^{-1}\right)}_{\vec{w}} \vec{y}$

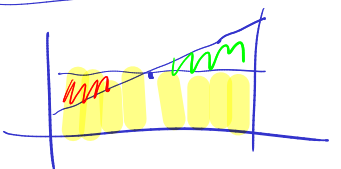
"interpolatory" "unbek. coeff."

$\vec{n} = \begin{pmatrix} \int_a^b 1\,dx \\ \int_a^b x\,dx \\ \int_a^b x^{n-1}\,dx \end{pmatrix} = \begin{pmatrix} w_1 \cdot 1 + \dots + w_n \cdot 1 \\ w_1 x_1 + \dots + w_n x_n \\ w_1 x_1^{n-1} + \dots + w_n x_n^{n-1} \end{pmatrix} \Leftrightarrow V^T \vec{w} = \vec{n}$

V^T

Equispaced: " Newton-Cotes"

Chebyshev: " Clenshaw-Curtis"



N-C
+ odd # nodes
+ symmetric

Gains degree

n points ↴

Interpolation : $\|f - p_{n+1}\|_\infty = O(h^n)$

Quadrature : $|Sp - p_{n+1}\|_\infty = O(h^{n+1})$

could be increased by gains degree

	n	Deg.	Ex.Int.Deg. (w/odd)	Intp.Ord.	Quad.Ord. (regular)	Quad.Ord. (w/odd)
		$n-1$	$(n-1)+1_{\text{odd}}$	n	$n+1$	$(n+1)+1_{\text{odd}}$
Midp.	1	0	1	1	2	3
Trapz.	2	1	1	2	3	3
Simps.	3	2	3	3	4	5
S. 3/8	4	3	3	4	5	5

$$\left|\sum w_i f(x_i) - \sum w_i \tilde{f}(x_i)\right| \leq \sum |w_i|\, \varepsilon(x_i)$$

$$\|\varepsilon\|_\infty \sum |w_i| \qquad \text{with neg. weights: } \sum |w_i| \gg \int_a^b 1\,dx$$

'Fair' $\|\alpha\|_\infty \cdot \int_0^b 2\,dx$