March 11, 2026

Announcements

- Exam 2 second ends today
- HW4 due tonight
- HW5 "technically" over break

Goals

- Secant
- "Wild" methods
- n dimensions
  - FPI
  - Newton
  - Broyden
  - AD / Testing derivatives

Secant

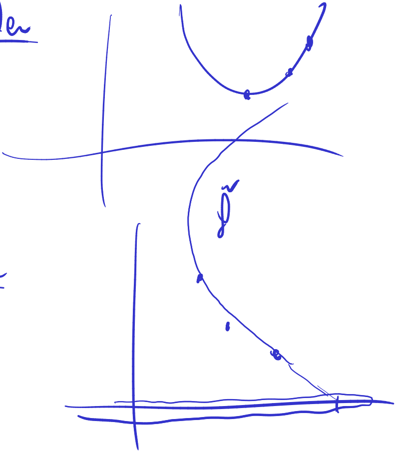
	Newton	Secant
Cost per iteration	$2f$	$1f$
Rate	2	1.618

$$e_{k+1} \lesssim e_{k+1}^{r} \lesssim \left(e_k^{r}\right)^{r} = e_k^{r^2}$$

$$e_{k+1} \leq C \underbrace{e_k^{r-1}}_{\uparrow} e_k$$

Muller

$$\frac{f(x)}{(x-x^*)}$$



IQI

$$\tilde{f}_{IQI} : y \mapsto x$$

$$(x_i, y_i)$$

$$x_{k+1} = \tilde{f}_{IQI,k}(0)$$

Fixed-point iteration

$$\vec{x}_{n+1} = \vec{g}(\vec{x}_n) \qquad \vec{g}(\vec{x}^*) = \vec{x}^*$$

$$\vec{e}_n := \vec{x}_n - \vec{x}^*$$

$$\vec{e}_{n+1} = \vec{x}_{n+1} - \vec{x}^* = \vec{g}(\vec{x}_n) - g(\vec{x}^*)$$

$$= J_g(\vec{x}^*) \cdot \vec{e}_n + O(\|e_n\|^2)$$

$$\|e_{n+1}\| = \|J_g(\vec{x}^*)\,\vec{e}_n\| \leq \|J_g(\vec{x}^*)\|\;\|\vec{e}_n\|$$

$$\rho(J_f(x^*)) \qquad \rho(J_g(x^*)) < 1$$

Generic Taylor:

$$\vec{f}(\vec{x}+\vec{h}) = \vec{f}(\vec{x}) + \partial_{f}(\vec{x}) \cdot \vec{h} + O(\|h\|^2)$$

Newton

$\vec{x}_0 = \langle \text{initial guess} \rangle$

$$\vec{f}_k(\vec{x}_n + \vec{h}) = \vec{f}(\vec{x}_n) + J_f(\vec{x}_n)\vec{h}$$

$$\vec{x}_{n+1} = \vec{x}_n + \vec{h}$$

$$= \vec{x}_n - J_f^{-1}(x_n)\,\vec{f}(\vec{x}_n)$$

n^2 entries

$$J_f(\vec{x}_n)\,\vec{h} = -\vec{f}(\vec{x}_n)$$

$$\vec{h} = -J_f(\vec{x}_n) / \vec{f}(x_n)$$

Broyden

$$x_{n+1} = x_n - \text{"}\underset{\substack{\uparrow \\ n^2}}{\text{Jacobian}}\text{"}\, f(x_n)$$

$$\tilde{J}\left(\vec{x}_{n+1} - \vec{x}_n\right) = f\left(\vec{x}_{n+1}\right) - f\left(\vec{x}_n\right)$$

Testing derivatives

$\vec{s} \leftarrow$ pick randomly

$$\lim_{h \to 0} \left\| \frac{\vec{f}(\vec{x} + h\vec{s}) - f(\vec{x})}{h} - \partial_f(x)\,\vec{s} \right\|$$

$$\frac{f(x+h) - f(x)}{h}$$