Today

0. projeqt ϕ due ... "idea"

1. projet I due Friday

2. Code 2D FE. assembly with linear basis.

Where are we at?

Find $u^h \in V^h$ = space of linears on each triangle + continuous

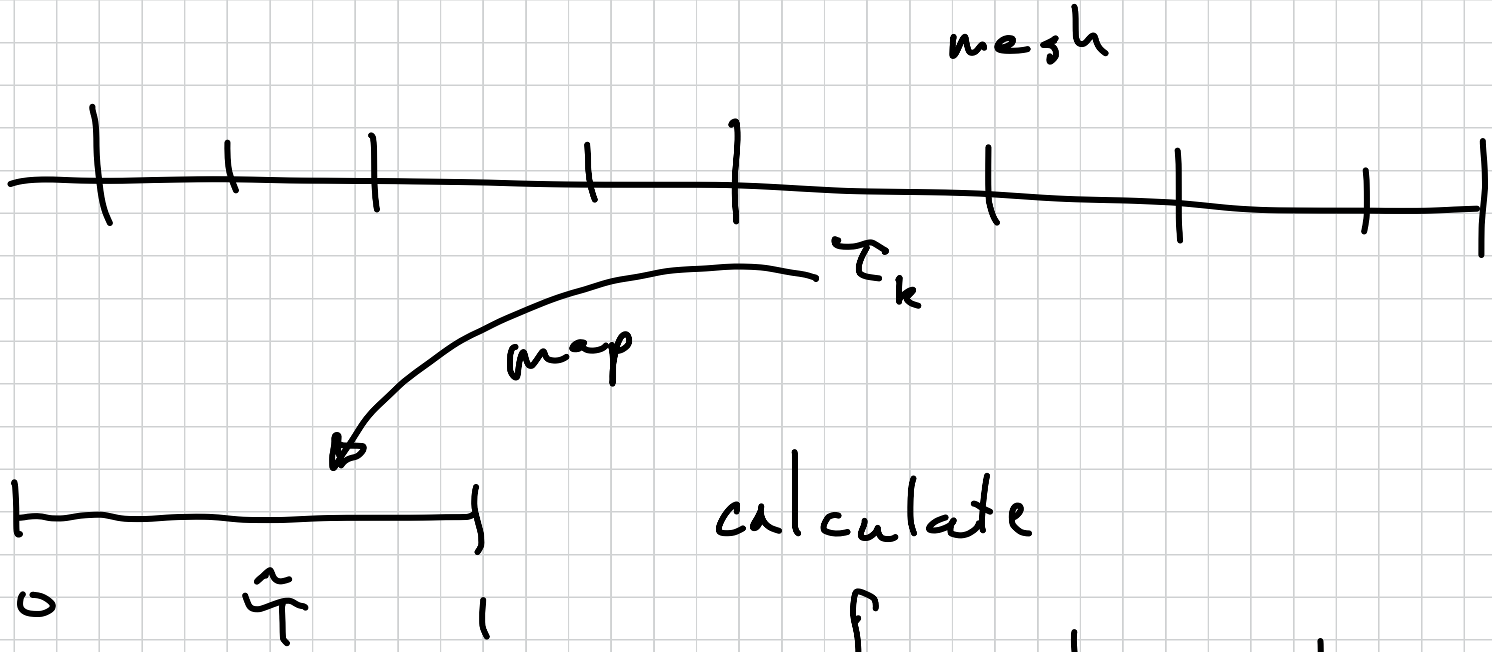
s.t.

$$\int_\Omega \nabla u^h \cdot \nabla \phi_j \, dx = \int_\Omega f \, \phi_j \, dx$$

$\forall \; \phi_j \in$ basis for V^h

let $u^h = \sum_{i=1}^{n} c_i \phi_i$

1D :



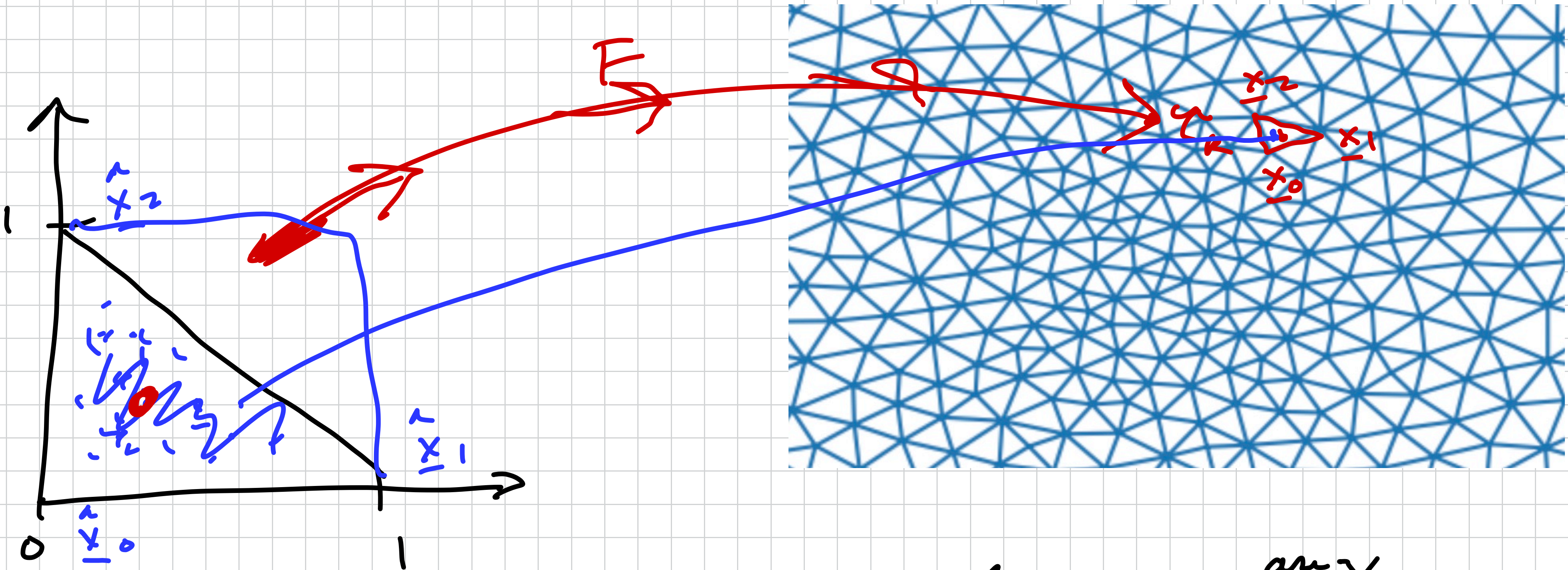
calculate

$$\int_{\hat{\tau}_k} \nabla \phi_j \cdot \nabla \phi_i \, dx$$

over

$$\int_0^1$$

using change of variables.



What is F? $\quad F : \hat{\tau} \longrightarrow \underset{\text{any}}{\tau_K}$

$$F(\underline{x}) = \hat{C} \cdot \underset{\hat{\tau}}{\underline{\hat{x}}} + \hat{\underline{b}}$$

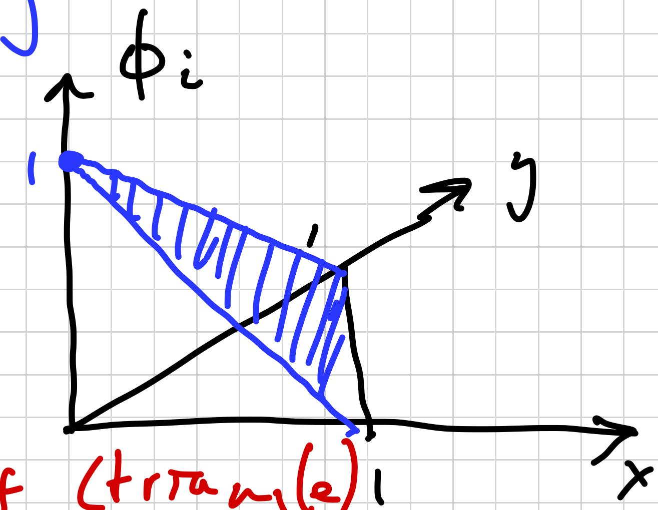
$\hat{\tau}$ coordinates in $\hat{\tau}$

$$= \begin{bmatrix} x_1 - x_0 & x_2 - x_0 \\ y_1 - y_0 & y_2 - y_0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\text{Jacobian of } F = \begin{bmatrix} x_1 - x_0 & x_2 - x_0 \\ y_1 - y_0 & y_2 - y_0 \end{bmatrix}$$

Have a basis on $\hat{\tau}$:

$$\hat{\phi}_0(\hat{\underline{x}}) = 1 - \hat{x} - \hat{y}$$
$$\hat{\phi}_1(\hat{\underline{x}}) = \hat{x}$$
$$\hat{\phi}_2(\hat{\underline{x}}) = \hat{y}$$



let $\underline{x} \in \tau$ ← any element (triangle)

then $F^{-1}(\underline{x}) \in \hat{\tau}$

so $\phi_i = \hat{\phi}_i(F^{-1}(\underline{x}))$ is linear in $\hat{\tau}$

$$\Rightarrow \nabla \phi_i = \nabla\left(\hat{\phi}_i(F^{-1}(x))\right)$$

chain rule

$$= J_F^{-T} \nabla_{\hat{x}} \hat{\phi}_i(F^{-1}(x))$$

want $\displaystyle\int_{\tau} K(\underline{x})\, \nabla\phi_j \cdot \nabla\phi_i \, dx$

any $\rightarrow \tau$

$$= \int_{\hat{\tau}} K(F(\hat{x}))\, J_F^{-T} \nabla_{\hat{x}} \hat{\phi}_j \cdot J_F^{-T} \nabla_{\hat{x}} \hat{\phi}_i \, |J_F|\, d\hat{x}$$

ref. $\rightarrow \hat{\tau}$

also $\displaystyle\int_{\tau} f(\underline{x})\, \phi_i(\underline{x})\, dx$

any $\rightarrow \tau$

$$= \int_{\hat{\tau}} f(F(\hat{x}))\, \hat{\phi}_i \, |J_F| \, dx$$

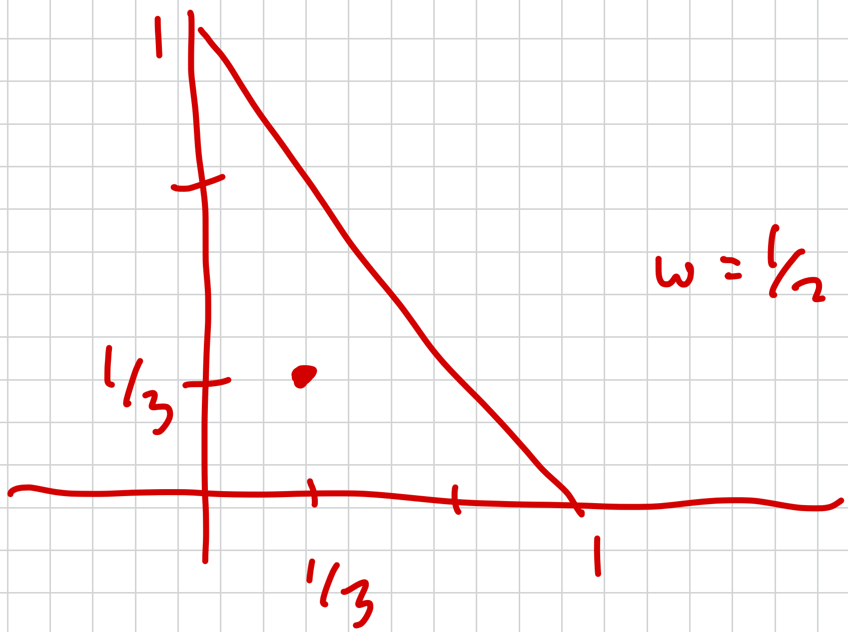
ref $\rightarrow \hat{\tau}$

$$\hat{\phi}_0 = 1 - \hat{x} - \hat{y} \Rightarrow \nabla_{\hat{\underline{x}}} \hat{\phi}_0 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\hat{\phi}_1 = \hat{x} \Rightarrow \nabla_{\hat{\underline{x}}} \hat{\phi}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\hat{\phi}_2 = \hat{y} \Rightarrow \nabla_{\hat{\underline{x}}} \hat{\phi}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\left(\tfrac{1}{3}, \tfrac{1}{3}\right)$$



$w = 1/2$