CS556—Iterative Methods: Tentative Schedule, Fall 2024

Week 1–2:

- Course Overview
- Introduction / Examples
  - A first iterative example
  - A second iterative example: 1D Poisson
  - Grid-function evaluation as matrix-vector products
  - Stencils
  - Kronecker Product Introduction
- Gaussian elimination
  - The Geometry of Linear Systems
  - Full systems
  - Block Methods
  - Banded systems
- Saad Chapter 1, Linear Algebra Basics
  - Norms
  - Diagonally dominant matrices, irreducibly diagonally dominant
  - M matrices
  - Eigenvalues / eigenvectors / Schur form, etc.
  - Projection: 1D / n-D

Week 2–3:

- Saad Chapter 2: Discretization of PDEs
  - Finite differences
    - Poisson / Helmholtz / Advection-diffusion 1D
    - Poisson 2D
    - Poisson 3D
    - Eigenvalues
  - Finite elements
    - Poisson 2D / 3D
- Direct Methods and the Curse of Dimensionality
- Fast Poisson Solvers using Kronecker products
- HPC considerations (Saad Chapter 11.2)
  - Pipelined/vectorized arithmetic
  - Memory hierarchies (caches)
  - Interpretive languages
  - Examples that slow performance
  - Examples with high performance

Week 4:

- Sparse Matrices
  - Graph representations
  - Sparse-matrix formats (CSR, etc.)
- Reordering and Sparse Direct Methods
  - Minimum degree ordering
  - Nested dissection ordering
  - A-conjugacy of ND orderings
  - Impact of reordering on matrix fill

Week 4–5: Basic Iterative Methods

- Jacobi, GS, SOR
- Jacobi vs GS: Poisson v. Advection-Diffusion
- Some convergence results
- ADI: Poisson / Helmholtz

Week 6-7: Projection Methods

Week 6: Conjugate Gradient Iteration

- Derivation and convergence rate
- Unpreconditioned variant
- Preconditioned variant (inc. Jacobi PCG example)
- Relationship to orthogonal polynomials
- Relationship to Steepest Descent
- Relationship to Lanczos iteration for eigenproblems
  - e.g., solving $f(A)\underline{x} = \underline{b}$
- Flexible CG

Week 7: GMRES

- Full GMRES
- Restarted GMRES
- Flexible GMRES
- Left/Right preconditioning
- Alternatives to GMRES (Saad Chapter 7)

Week 8–11: Preconditioning

- Block Jacobi
- Overlapping Schwarz (additive/multiplicative)
- Multilevel Schwarz
- Substructuring
- ILU

Week 11–13: Multigrid

Remark. We will introduce many of the topics throughout the course with the intent that the principal discussion and exploration of a given topic be covered in the proposed timeline. The overall aim of the course is for students to be familiar with a broad range of tools to efficiently solve large linear systems. Choosing the correct solver can save orders-of-magnitude in computational costs, so coverage beyond iterative methods is important. In particular, the choice of optimal preconditioning strategies warrants familiarity with multiple solution algebra methods.