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LEC 7:

Last Time:

Iters Jacobi $\sim 2.5N^2$

GS $\sim \frac{1}{2}(2.5N^2)$

$\underline{S}OR_{opt} \sim 2N$ (Chan Elman '88)

unpreconditioned CG

$$err = \frac{\|e_k\|_A}{\|\underline{x}\|_A} \le 2\left(\frac{\sqrt{K}-1}{\sqrt{K}+1}\right)^k \sim 10^{-6}$$

$\epsilon := \frac{1}{\sqrt{K}}$

$K = \frac{4N^2}{\pi^2}$

$\sqrt{K} = \frac{2N}{\pi}$

$$(1-\epsilon)(1+\epsilon)^{-1} = 1 - 2\epsilon + O(\epsilon^2)$$

$$(1-2\epsilon)^k = \frac{1}{2}10^{-6}$$

$$e^{-2\epsilon k} = \frac{1}{2}e^{-12}$$

$$k \simeq \frac{12}{2\epsilon} = 6\epsilon^{-1} \sim \frac{6 \cdot 2}{3} N \sim 4N$$

①

- Derivation of CG:

- Small detour: Richardson Iteration

- For the case where $D(A) = a_{11} I$, Jacobi iteration becomes

$$\underline{x}_{k+1} = \underline{x}_k + D^{-1}(\underline{b} - A x_k)$$

$$= \underline{x}_k + \alpha(\underline{b} - A\underline{x}_k) \qquad \alpha = \frac{1}{a_{11}} = \frac{h^2}{2d}$$

- $$\underline{e}_k = \underbrace{(I - \alpha A)}_{G} \underline{e}_{k-1}$$

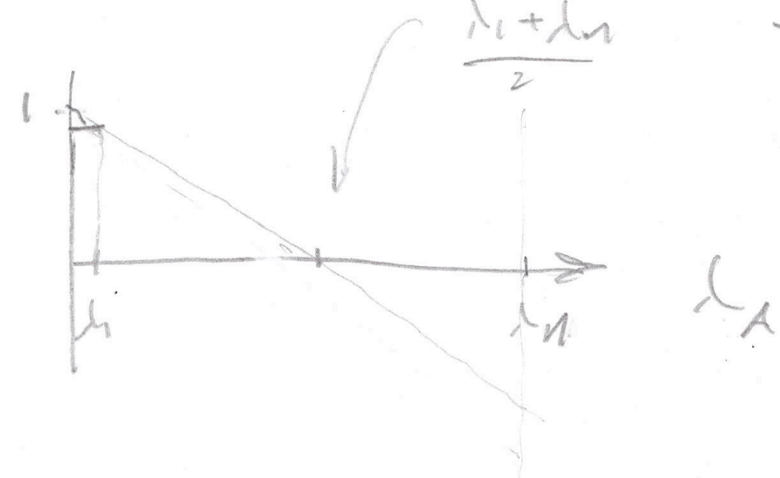
- Require $\rho(G) < 1$

③

- Eqn 4 — $H \to \frac{1}{h^2} H$

- Upload Link Q HW

$g(\lambda) = 1 - \alpha\lambda$



$$\alpha = \left(\frac{2}{\lambda_1 + \lambda_n} \right)$$

- If $Re(\lambda_A) > 0$
  Can always find α s.t. Richardson converges

why?

Richardson analogous to EE timestepping:

u^k —

$$\frac{du}{dt} = L\underline{u} + \underline{f} \qquad (L = -A \text{ in our context})$$

$$\frac{du}{dt} \approx \frac{u^{k+1} - u^k}{\Delta t} = L\underline{u}^k + \underline{f}$$

③ If Steady State exists use $Re(\lambda(L))<0$

As $t \to \infty$ $\quad \frac{du}{dt} \to 0 \quad \underline{u} \to \underline{u}^\infty$

$$L\underline{u}^\infty + \underline{f} = 0 \qquad -L\underline{u}^\infty = \underline{f}$$

$$A\underline{u}^\infty = \underline{f}$$

$$A\underline{u} = \underline{f}$$

EF

$$\frac{\underline{u}^{k+1} - \underline{u}^k}{\Delta t} = \underline{L}\,\underline{u}^k + \underline{f}$$

$$u^{k+1} = \underline{u}^k + \Delta t L \underline{u}^k + \Delta t \underline{f}$$

$$u^\infty = u^\infty + \Delta t L \underline{u}^\infty + \Delta t \underline{f}$$

$$\underline{e}^{k+1} = \underbrace{(I + \Delta t L)}_{G(L)} \underline{e}^k$$

model problem $\quad L = \lambda(L) =: \lambda_L$

$$|g(\lambda_L)| = |1 + \Delta t \lambda_L| \le 1$$

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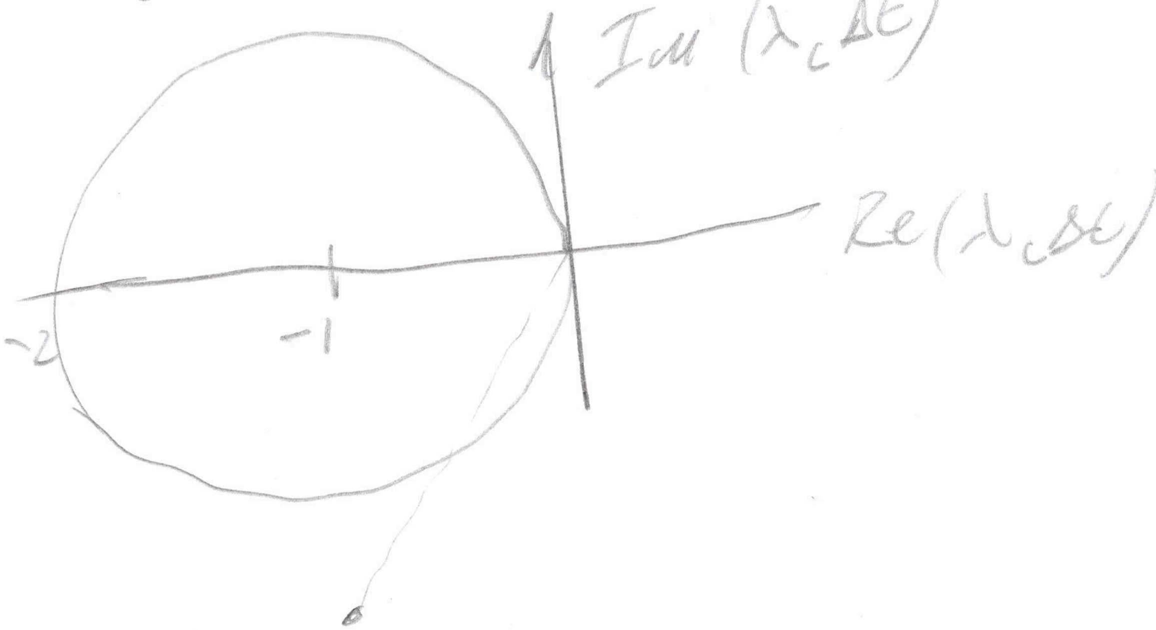

If Re$(\lambda_c) < 0$
Can always find a Δt s.t. $|g(\lambda_c)| < 1$