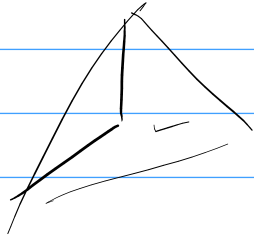
Today

~~HW2~~

- rank mystery
- multipole expansion
- expansion using Ideal (μ|)
- fast alg.

Error in TE of potential $\leq C \left(\dfrac{d(c, \text{furthest tgt})}{d(c, \text{closest source})} \right)^{k+1}$

Num terms in 3D Taylor exp of order k

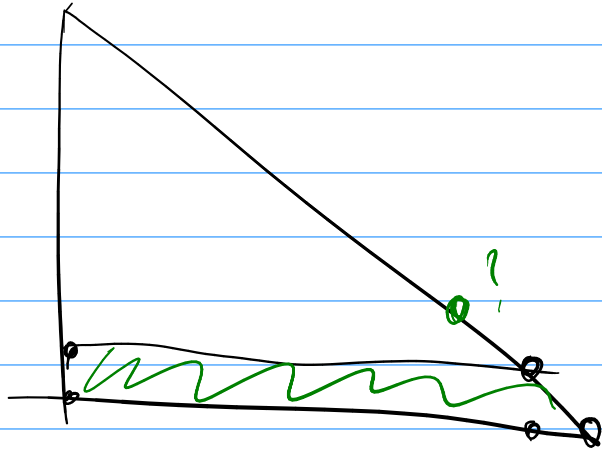


$$= O(k^3)$$

$$\frac{(k+1)(k+2)(k+3)}{6}$$

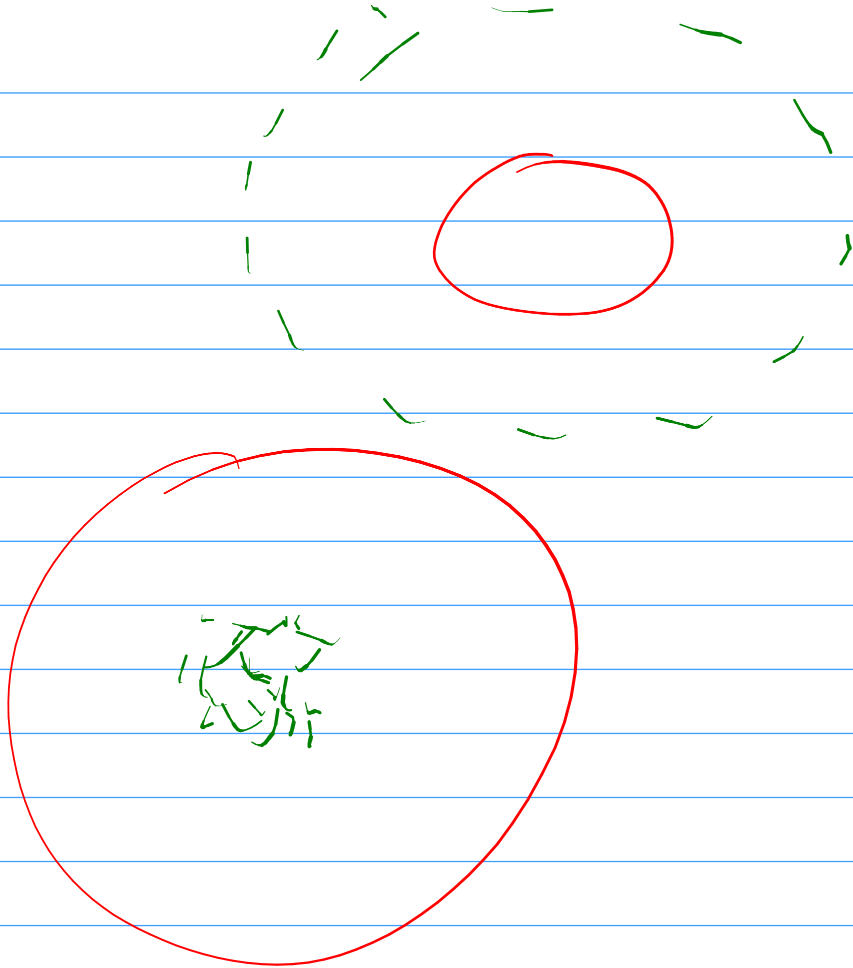
$$\text{num terms} \approx \left(\frac{\log \varepsilon}{\log \rho} - 1 \right)^2$$

$$\rho = \frac{dfd}{dcs}$$



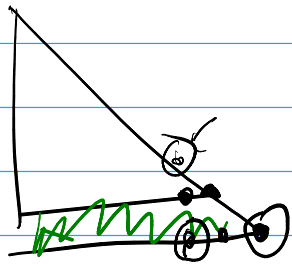
Have: $\partial_x^k u = \partial_x^{k-2} \partial_x^2 u$

Want: $\partial_x^{k-2} \partial_y^2 u$



$$(\Delta + k^2)u = 0$$

$$\partial_t^2 u = \Delta u \qquad u(t,\vec{x}) = u(\vec{x})e^{-i\omega t}$$



$$\partial_y^2 = (-\partial_x^2 - k^2)u$$

Taylor and Error

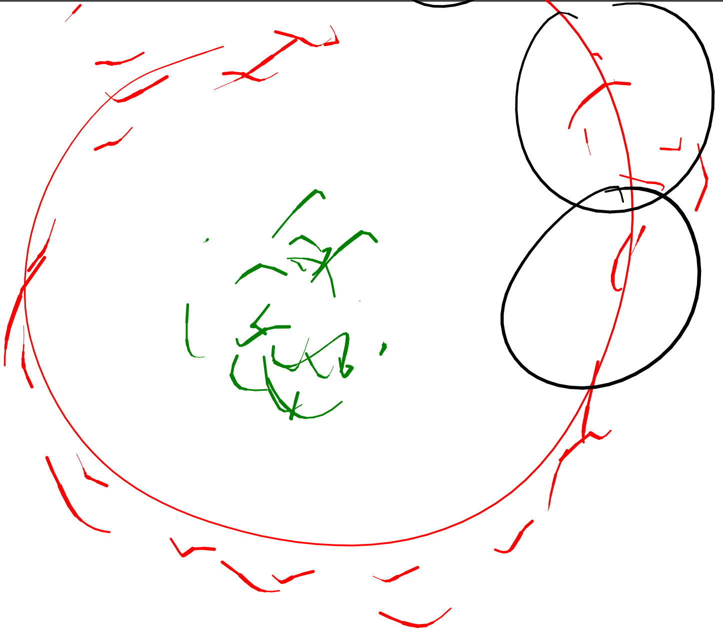
How can we estimate the error in a Taylor expansion?

Connect Taylor and Low Rank

Can Taylor help us establish low rank of an interaction?

Taylor on Potentials

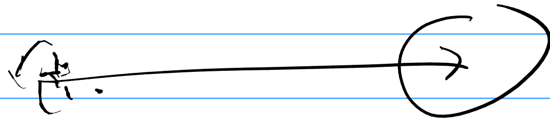
Compute a Taylor expansion of a 2D Laplace point potential.



Local: $$\psi(\vec{x}-\vec{y}) = \sum_{|\nu|\le k} \frac{D^{\nu}\psi(\vec{x}-\vec{y})\big|_{\vec{x}=\vec{c}}}{\nu!}(\vec{x}-\vec{c})^{\nu}$$

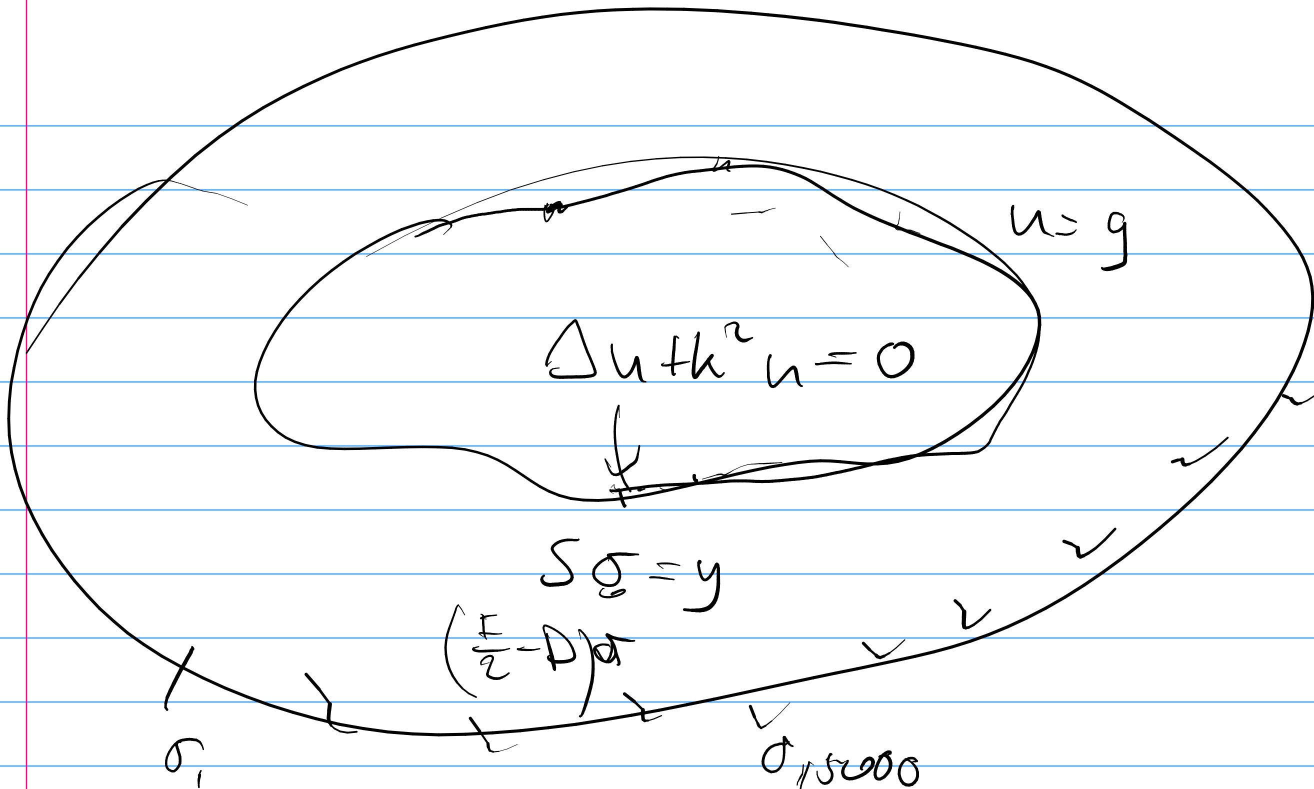
x: target
y: sources

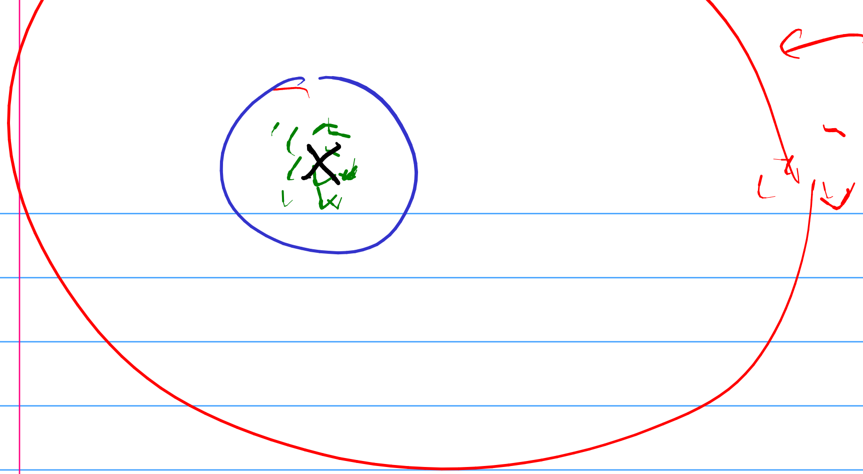
$$\text{Errors} \left(\frac{d_{ft}}{d_{cs}}\right)^{k+1}$$



Multipole: $$\psi(\vec{x}-\vec{y}) = \sum_{|\nu|\le k} \frac{D^{\nu}\psi(\vec{x}-\vec{y})\big|_{\vec{y}=\vec{c}}}{\nu!}(\vec{y}-\vec{c})^{\nu}$$

$$\text{Errors} \le \left(\frac{d_{fs}}{d_{ct}}\right)^{k+1}$$


$u = g$
$\Delta u + k^2 u = 0$
$S\sigma = y$
$\left(\frac{I}{2} - D\right)\sigma$
σ_1
σ_{15000}



targets good

Local expansions as a Computational Tool

Low rank makes evaluating interactions cheap(er). Do local expansions help with that goal?

Taylor on Potentials, Again

Stare at that Taylor formula again.

On Rank Estimates

So how many terms do we need for a given precision ε?

Estimated vs Actual Rank

Our rank estimate was off by a power of $\log \varepsilon$. What gives?

Being Clever about Expansions

How could one be clever about expansions? (i.e. give examples)