Today

- skeletonization
  - local
  - imp.
- Eval a summation
- Barnos-Hut
- Fast Multipole

- HW3 out
- HW2 rank finder;
  try to design incremental alg.

\[
\begin{align*}
A & \sim Q, \\
(A - Q, Q^\top A) \sim Q^2 \\
A & \sim Q, Q^\top A + Q^2 Q^\top A
\end{align*}
\]
Making Multipole/Local Expansions using Linear Algebra

Actual expansions seem vastly cheaper than LA approaches. Can this be fixed?

Local exp via proxies:

\[ S \]

\[ K \cdot T \]
\[ P \left( \text{Interaction}(T_{\text{Edg}}, S) \right) \]

\[ \begin{array}{c}
T \times k \\
\text{\textbf{5} x S}
\end{array} \]

\[ \begin{array}{c}
( k \\
\text{\textbf{7}})
\end{array} \]
Multipole via proxies:
Why Does the Proxy Trick Work?

In particular, how general is this? Does this work for any kernel?
Where are we now?

Summarize what we know about interaction ranks.
Near and Far: Separating out High-Rank Interactions
Simple and Periodic: Ewald Summation

Want to evaluate potential from an infinite periodic grid of sources:

\[ \psi(x) = \sum_{i \in \mathbb{Z}^d} \sum_{j=1}^{N_{\text{src}}} G(x, y_j + i) \varphi(y_j) \]
Barnes-Hut: Putting Multipole Expansions to Work

(Figure credit: G. Martinsson, Boulder)

Want: All-pairs interaction.
Caution: In these (stolen) figures: targets sources.
Here: targets and sources.
Barnes-Hut: Putting Multipole Expansions to Work

(Figure credit: G. Martinsson, Boulder)
Barnes-Hut: Putting Multipole Expansions to Work

(Figure credit: G. Martinsson, Boulder)

For sake of discussion, choose one ‘box’ as targets.

**Q:** For which boxes can we then use multipole expansions?
Barnes-Hut: Putting Multipole Expansions to Work

(Figure credit: G. Martinsson, Boulder)

With this computational outline, what’s the accuracy?